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# Mathematical foundations of and empirical investigations into the dynamic of top positions: Stabilization Effect, Reversed Matthew Effect, and Heraclitus Effect

*Albert Ziegler jr.*[1], *Annika Heckel*[2] & *Albert Ziegler*[3]

**Abstract**

The performance differences between successively ranked individuals tend to increase towards the top. However, the mathematical foundations of this effect are still largely untapped. This article will focus on developing such foundations. It will also be shown that the effect is stable for various natural distributions of eminent achievements. Three new predictions about the dynamics of top positions are formulated and tested with two samples from the world of sports: the best male chess players (individual sport) and male national soccer teams. The stabilization effect describes the phenomenon that the stability of ranks is higher among the top ranks. The reversed Matthew effect asserts that achievement gains among elite players and elite teams are positively correlated with their ranks (i.e. diminishing towards the top). However, in contrast, the Heraclitus effect predicts that the performance gains among the top ranks are nevertheless bigger than what can be mathematically expected from the position in the ranking. All three effects can be empirically corroborated.

Keywords: Top positions, Eminence, Stabilization Effect, Reversed Matthew Effect, Heraclitus Effect

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[1] *Correspondence concerning this article should be addressed to:* Albert Ziegler, PhD, School of Mathematics, University of Leeds, Leeds, LS2 9JT, UK; email: mmaz@leeds.ac.uk

[2] University of Oxford

[3] University of Erlangen-Nuremberg

Following Francis Galton (1849), human attributes have long been seen as generally normally distributed according to the Gaussian bell curve, with most people found in the medium range. Leaving this range, the number of people dwindles quickly until it asymptotically tends to zero. Galton defined the concept of giftedness by distinguishing those who are located in the rightmost part of the bell curve, however, he did not specify an exact cut-off point. Terman (1925), who contributed the first large longitudinal study to the research of giftedness, advocated taking the highest 1% according to intelligence.

While the distribution of intelligence displays a high degree of symmetry between very high and very low intelligence, there is a marked asymmetry between very high and medium intelligence that has often given cause to rethink the question of top performance.

Indeed for a long time in history, it was commonly assumed that a few eminent persons would disproportionately contribute to top achievements while the majority would rarely do so or even not at all.

This notion is often traced back to the Greek philosopher Heraclitus of Ephesus (c. 535 BC – c. 475 BC) who famously said, “One man is worth ten thousand others if he is the best.” More recently, Simonton (1984, 1988, 1997, 1999, 2003) established in various studies that the differences in the level of performance between successive ranks increase the closer one gets to the very top. He stressed that top achievements are not at all distributed like a bell curve but that a few excellent achievers contribute a disproportionately large part of all high achievements (Simonton, 2004, 2009a). Systematic evidence for this was first collected by Dennis (1954a, 1954b, 1955). Additionally, there are several obvious examples, even in everyday life. For instance, there is a large number of directors producing good movies, but among the very best of movies there are some directors’ names which recur particularly often. Similarly, there is an immense number of authors but some of them appear several times when only considering the greatest novels. Scientific confirmation of these everyday observations can be found in the literature (see Huber, 2000; Murray, 2003; Walberg, Strykowski, Rovai, & Hung, 1984).

Following Simonton, we propose that the current state of research on the contribution of eminent persons to top achievements can be summarized in five statements (Simonton, 2000, 2004, 2009a, 2009b):

1. The differences in performance between successive achievers or groups of achievers increase towards the top.
2. Only a few persons are responsible for a large proportion of top achievements.
3. As the number of achievements increases, the importance of the top achievers increases also, as the majority of top achievements is accomplished by a smaller percentage.
4. As the number of people accomplishing top achievements increases, the importance of the very top achievers also increases because, within the elite, another sub-elite forms.
5. The number of achievements accomplished by one individual correlates positively with their quality.

Attempts to explain the disproportionately high contributions of eminent persons have mostly relied on intuitive assumptions about mathematical distributions (Simonton, 1999, 2004, 2009a, 2009b). For instance, the Pareto principle, also known as the law of the vital few, claims that, in many areas, the majority of valuable output is produced by a relatively small group of high achievers (for example, the eponymous Vilfredo Pareto observed that the highest yielding 20% of the peapods in his garden produced 80% of all his peas). This article will first investigate the intuitive mathematical assumptions in a more systematic way, based on the empirically-confirmed findings about differences between successive ranks increasing towards the top. Building on this mathematical analysis, we will produce several predictions about the dynamics of top ranks, which to the best of our knowledge are new. We will then proceed to test them empirically.

## Aims of the current research

This contribution consists of theoretical ($H_1$ to $H_4$) and empirical ($H_5$ to $H_9$) considerations, as follows.

$H_1$: While it has often been noted that the differences between exceptional achievers are quite small (e.g., Simonton, 2000), a purely statistical effect forces them to actually be quite a bit larger than the differences between average achievers.

$H_2$: Hypothesis $H_1$ applies to several natural distributions, but, in particular we will consider normal distributions, Pareto distributions and Poisson distributions.

$H_3$: Hypotheses $H_1$ and $H_2$ imply an increased reliability of the measurement of eminent achievements.

$H_4$: This hypothesis centers on a concept closely related to reliability, namely stability over time. The stability will be shown to be higher for eminent achievements than for average achievements. We will expand on this insight in hypotheses $H_7$ to $H_9$, which describe new assumptions about the dynamic of top positions.

The empirical significance of the mathematical considerations will be demonstrated with two samples of data from different areas of sport. As an example for an individual sport, we review the world's best chess masters, and, as an example for a team sport, we review the top national soccer teams.

$H_5$: Ratings and ranks at the very top are stable over the course of a year. This is a prerequisite to investigate the predicted effects from $H_7$ to $H_9$ for the samples of chess players and soccer teams.

$H_6$: The difference in performance for two successive ranks increases towards the top, both for the individual sport of chess and for the team sport of soccer. This is the empirical test of the theoretically obtained $H_1$ and, together with $H_5$, also forms the empirical basis for the investigation of $H_7$ to $H_9$.

$H_7$: Stabilization effect. At better positions in the global ranking, the stability increases. This is the empirical version of the statistical effect obtained in $H_5$ and will be tested both for chess players and for soccer teams.

$H_8$: Reversed Matthew effect. At first glance, one might speculate that the increasing differences at better ranks imply that the increases in performance are also higher at the top. This would lead to a performance explosion, however, where the very best continuously and quickly keep getting better and develop an ever increasing gap with all others. Yet this would bring about some absurd consequences, such as the best sprinters tending towards the speed of light. This is why a deacceleration mechanism is proposed, whereby better rankings are connected to lower performance increases in the future. This prediction aligns with well documented findings in the field of skill acquisition research, according to which on an individual level, performance increases initially happen very quickly and then slowly converge to an asymptote (Heathcote, Brown, & Mewhort, 2000; Logan, 1992; Newell & Rosenbloom, 1981). Those athletes or teams positioned near the top ranks can be expected to be closer to the ceiling and thus to show a smaller increase in performance. This directly contradicts another effect, the Matthew effect, which states that those who already have a head start will increase this start over time (Rigney, 2010). We believe that our findings show that eminent achievements belong to an area where the Matthew effect does not apply. However, we do predict a related effect.

$H_9$: Heraclitus effect. The reversed Matthew effect ensures that the performance of top ranks does not tend to infinity. Nevertheless, we still believe that there are advantages to holding a top ranking. It is natural to assume that top positions lead to an easier accessibility of resources, which in turn can be used to maintain and improve performance (cf. Ziegler & Baker, 2013). For example, a better relative position is usually rewarded with an increase in Educational Capital. These financial resources can be invested in better coaches or better training infrastructure, which leads to an added advantage over less established competitors. The reversed Mathew effect implies that performance increases are lower at the top positions. The Heraclitus effect now claims that the improvements are still higher than should be expected by the reversed Matthew effect alone. In other words, while it is impossible for those already placed at top positions to improve as much as those of lower ranks, they still improve over par.

## Theoretical considerations

### Greater differences at exceptional values

While it has often been noted that the differences between exceptional achievers are quite small, a purely statistical effect forces them to actually be quite a bit larger than the differences between regular achievers.

Consider $n$ independent random variables $X_1, \ldots, X_n$, which describe the absolute level of performance or aptitude of $n$ individuals, such as the Elo ratings of $n$ chess players or test scores of $n$ students. We assume these random variables have the same distribution with cumulative distribution function $F(x)$, which is given by the probability that the value of one random variable, say $X_1$ , is at most $x$. We assume that $F(x)$ is differentiable; its derivative $f(x) = F'(x)$ is then the probability density function of the distribution.

We additionally assume that the random variables $X_1, \ldots, X_n$ only take values that are greater than some minimal value $M > 0$, that the density function $f(x)$ is 0 for all $x < M$, and that $f(x)$ is *strictly decreasing* for $x \geqslant M$, so that $f(x) > f(y)$ whenever $M \leqslant x < y$. This is a valid assumption since the density functions of most natural distributions are strictly decreasing in the extremal range, where the absolute values of individuals with extreme ranks can be found.

We now order the random variables by size. Let $m_1$ be the index of the largest random variable $Xm_1$, corresponding to the individual ranked first. Let $m_2$ be the index of the second largest random variable $Xm_2$ and so forth, so that $Xm_1 > Xm_2 > ... > Xm_n$. The numbers $m_k$ are of course random variables themselves. Note that with probability 1, no two different random variables take the same value because we assumed that their distribution has a probability density function $f(x)$.

We are interested in the difference between the first ranked and the second ranked individual $Xm_1 - Xm_2$ or more generally between the individuals with ranks $k$ and $k + 1$, $Xm_k - Xm_{k+1}$. It turns out that the lower the rank, the more likely there are large performance differences of adjacent ranks. The probability that the performance difference is larger than any given constant decreases as the rank gets larger.

**Claim:** For every $k = 1, ..., n - 2$, and every $c > 0$, $P[Xm_{k+1} - Xm_k > c] > P[Xm_{k+2} - Xm_{k+1} > c]$ **.**

**Proof:** There are $n$ possibilities which of the random variables $X_1, \ldots, X_n$ has rank $k$, that is, for which $i$ we have $X_i = X\, m_k$ (by our assumptions on the distribution, almost surely, i.e. with probability 1, no two different random variables take the same value).

If $Xi = x$ for some $x \geqslant 0$, then $Xm_{k+1} - Xm_k > c$ with $X_i = Xm_k$ happens if and only if $k$ - 1 of the $n$ - 1 other random variables are larger than $x$, and the remaining $n-k$ random variables are less than $x - c$. Since all the random variables have the same cumulative distribution function $F(x)$, this event has probability $\binom{n-1}{k-1} (1-F(x))^{k-1} F(x-c)^{n-k}$.

Taking the integral over all possible values $x$ such that $X_i = x$, and summing over all $n$ possible values $i$ so that $X_i = Xm_k$, this gives

$$P\left[X_{m_{k+1}} - X_{m_k} > c\right] = \int_M^\infty n\binom{n-1}{k-1}\left(1-F(x)\right)^{k-1} F(x-c)^{n-k} f(x)dx \qquad (1)$$

Since $f(x) = F'(x)$, we can use integration by parts – note that

$$\int (1-F(x))^{k-1} F(x-c)^{n-k} f(x)dx = -\frac{1}{k}\cdot(1-F(x))^k F(x-c)^{n-k}$$
$$+\int \frac{n-k}{k}\cdot\left(1-F(x)\right)^k F(x-c)^{n-k-1} f(x-c).$$

This gives

$$P\left[X_{m_{k+1}} - X_{m_k} > c\right] = \left[-\frac{n}{k} \cdot \binom{n-1}{k-1}\left(1-F\left(x\right)\right)^k F\left(x-c\right)^{n-k}\right]_M^\infty + \int_0^\infty \frac{n\left(n-k\right)}{k} \cdot \binom{n-1}{k-1}\left(1-F\left(x\right)\right)^k F\left(x-c\right)^{n-k-1} f\left(x-c\right)dx.$$

As $\lim_{x\to\infty} F(x)=1$ and $F(M) = 0$, the first part of the right-hand side is 0. Furthermore,

$$\frac{\left(n-k\right)}{k} \cdot \binom{n-1}{k-1} = \frac{\left(n-k\right)}{k} \cdot \frac{\left(n-1\right)!}{\left(k-1\right)!\left(n-k\right)!} = \frac{\left(n-1\right)!}{k!\left(n-1-k\right)!} = \binom{n-1}{k}.$$

Hence, we have

$$P\left[X_{m_{k+1}} - X_{m_k} > c\right] = \int_0^\infty n\binom{n-1}{k}\left(1-F\left(x\right)\right)^k F\left(x-c\right)^{n-k-1} f(x-c)dx.$$

Since $f(x)$ is strictly decreasing, $f(x-c) > f(x)$, so

$$P\left[X_{m_{k+1}} - X_{m_k} > c\right] = \int_0^\infty n\binom{n-1}{k}\left(1-F\left(x\right)\right)^k F\left(x-c\right)^{n-k-1} f(x)dx.$$

But the last integral is just the formula for $P\,[Xm_{k+2} - Xm_{k+1} > c]$ given by equation (1), with $k+1$ substituted for $k$. So this is just exactly $P\,[Xm_{k+1} - Xm_k > c] > P\,[Xm_{k+2} - Xm_{k+1} > c]$, as required.

***Note:*** *The differentiability of F(x) is not strictly necessary. Instead, the claim also holds with the weaker assumption that the cumulative probability function F(x) is strictly concave (for x⩾M). This implies the existence of a strictly decreasing probability density function f(x) with f(x)= F'(x) for almost every x. The claim can also be proved in this case with a slightly different strategy.*

## Application to natural distributions

The results from the last section are general in the sense that they show that increased differences at exceptional values are a direct consequence of fairly general assumptions on the probability distribution. $H_9$ states that most specific probability distributions that are used for statistical purposes do fulfill the following assumptions.

Normal distributions, which are used to approximate a vast variety of real world distributions, such as for intelligence, are characterized by the density function $e^{-(x-\mu)2/2\sigma 2}/\sigma\sqrt{2\pi}$, which means that roughly the upper half of the public (and, in particular, the range of high achievers) are distributed conforming to the demands of the last section with the lower half exhibiting the mirrored behavior. So differences are more pronounced for extremal values, be they particularly high or particularly low.

Poisson distributions are often encountered in cases where there is a natural lowest possible value. Given by $\lambda^x e{-}\lambda / x!$, they are discrete distributions. While the last section dealt with the real-valued case, the discrete case is simply easier and the results carry over (using sums instead of integrals). Thus, the differences begin to increase following the mode, which is the point after which the function $\lambda^x e{-}\lambda / x!$ is monotonously decreasing. For this particular distribution, this is close to the median and the mean, $\lambda$. In particular, for roughly the upper half of the values, an increase in the difference between successive ranks can be expected for higher values.

Pareto distributions are the distributions of entities conforming to Pareto's 80%-20% law and variations thereof (also known as power law distributions). As such they are of particular interest to the analysis of the distribution of achievement or output but also occur in many other fields. If the smallest possible value is 0, the density function of the Pareto distribution is of the form $\alpha(x+1)^{-\alpha-1}$ for $x \geq 0$ and 0 otherwise. This fulfills the conditions of the last section as it is monotonously decreasing for every value such that all neighborhoods of the value have positive probability. Thus, in these cases differences between adjacent values can always be expected to be higher for higher values. In the discrete case, this has distributions following Zipf's law as analogue.

Exponential distributions have a density of $\lambda e^{-\lambda x}$ for $x \geq 0$ and 0 otherwise. The same good properties as for the Pareto distributions hold and so differences can always be expected to be higher for higher values.

In fact most probability distributions that are being used in models fulfill the demands of the last section at least for large values. This means that in all these cases, the absolute differences between closely ranked high achievers will be larger than the differences between closely ranked medium-to-high achievers.

## Reliability of measurement

The effect of increased differences at extremal values has direct consequences for testing aimed at determining the rank order of subjects. Wherever the effect hits (and the previous sections demonstrate that this holds for the great majority of natural examples), it serves to increase the reliability of measurement at extremal values, which is what $H_3$ states.

Assume that the actual value is measured with a random error that is without systematic bias and independent of the true score (Crocker & Algina, 1986). If a random sample of people participate in the test and their scores are ranked ordinally, then for each person $i$ a ranking error $E_i$ can be calculated as the absolute value of the difference of their true rank and the rank that was measured. As according to our assumptions the difference between true value and measured value is independent of the value, the error $E_i$ can be expected to be larger if there are more persons in the sample with a value close to the true value of $i$. According to the results previously established, this means that for extreme values (where the differences between the values are larger), the error of measuring the rank $E_i$ will on average be smaller.

In practice, this statistical effect might of course be weakened if the error is not independent of the value, for example, if the test that was used is less reliable for extremal values to begin with as might be the case for tests that were developed for a medium value range only. Nevertheless, if the main focus is relative rankings, their measurement profits from increased reliability at extreme values due to the stability effect.

### Stability

The increased differences at extremal values directly lead to the prediction of another effect: $H_4$. If the value is tracked over time, the ranks of the test persons can be expected to be more stable at the very top.

This is due to a version of the same effect that also affects the reliability: the very top values can be expected to be further apart than those in less extreme ranges, thus if there is an unbiased random drift over time for the test persons' values independent of their current value, this will not result in swapped ranks at the top as often as it does near the middle.

Of course here this statistical effect might be weakened as well if there is a systematic relationship between the current value and its volatility. The empirical evidence, however, will show that such a relationship, if it exists, is usually eclipsed by the strength of the effect described in this section.

This is of particular importance as an individual's environment and its interactions often depend on the relative rank rather than the absolute value, whether in the microcosm of one class at school where the best student is treated differently than the average student or in the competitive world of athletes where the number of sponsors and the amount of funding depends on the athlete's relative performance. Thus this effect works at making the environments of the very top achievers more stable.

## Empirical evidence

The hypotheses $H_5$ to $H_9$ have been tested empirically on two samples: the individual sport of chess and the team sport of soccer. The sample for chess consists of the players ranked 1 to 100 in the global Elo rating system taken at July 2011 and July 2012. The sample for soccer also contains the global rankings, however, only the ranks 1 to 50. This restriction is justified as the focus of this study is top achievement. While several tens of thousands of players are represented in the chess rankings, the official soccer rankings include only the approximate 200 FIFA (International Federation of Association Football) members.

Chess and soccer were chosen because these two sports both measure performances with a sophisticated rating system. In the FIFA rating four factors are taken into account: match result, match status (e.g. friendly match or World Cup match), opponent´s strength, and regional strength. The rating is a result of the average points from the previous four years, with more weight being given to the recent ones.

In chess, the Elo ratings system is in use. It is considered one of the most valid measurements of performance across sport domains. In every game a player wins or loses points depending on the outcome of the game and the difference in the rating of the players. Every official game between rated players is considered.

*Testing* $H_5$: Relative rankings and absolute values of the ratings are stable for one year in the range of top achievers.

This hypothesis could be confirmed for both samples and in each case for ratings as well as rankings. For chess the correlation between the ratings in 2011 and 2012 was 0.84 ($p$<0.0001), and between ranks in 2011 and 2012 it was 0.75 ($p$<0.0001). Of the top 100 of 2011, 87 were still amongst the top 100 one year later.

Analysing the national soccer teams yields the same result. The correlation of ratings between 2011 and 2012 was 0.85 ($p$<0.0001) and for the ranks 0.80 (p<0.0001). Of the top 50 of 2011, 40 were still in the top 50 one year later.

*Testing* $H_6$: The differences between the ratings of successively ranked players or teams increase towards the top ranks.

In order to test this hypothesis, we calculated the correlations between the ranks and the margin between the rating and the rating of the player or team one ranking position lower. All correlations proved statistically significant in the predicted direction. Rank and rating margin for chess 2011: -.45 ($p$<0.001); Rank and rating margin for chess 2012: -.43 ($p$<0.001); Rank and rating margin for soccer 2011: -.54 ($p$<0.0001); Rank and rating margin for soccer 2012: -.26 ($p$<0.05)

*Testing* $H_7$: Stabilization effect.

To test this hypothesis, the top 100 chess ranks were partitioned into quartiles and the top 50 soccer ranks split into two parts along the median. The outcome can be found in Table 1. All results pointed in the predicted direction. The correlation of the first chess quartile (the top 25) was significantly higher than for any of the other subsamples (each $p$ was at least less than 0.05). For soccer, the differences between the correlations was only marginally significant in the ratings ($p$<0.10).

**Table 1:**
Stability of ranks and ratings for different subgroups between 2011 and 2012.

| Chess | | | Soccer | | |
|---|---|---|---|---|---|
| Subsample | Correlation | | Subsample | Correlation | |
| | Ranks | Ratings | | Ranks | Ratings |
| Rank 1-25 | 0.81*** | 0.83*** | Rank 1-25 | 0.72*** | 0.76*** |
| Rank 26-50 | 0.43* | 0.44* | Rank 26-50 | 0.50** | 0.49* |
| Rank 51-75 | 0.31 | 0.34 | | | |
| Rank 76-100 | 0.10 | 0.14 | | | |

Note: * = $p<0.05$; ** = $p<0.01$; *** = $p < 0.001$

*Testing* $H_8$: Reversed Matthew effect.

We predicted that those players and teams who were already placed higher in the global rankings would have a lower increase in their ratings than those ranked lower than them. This could be confirmed. In chess the correlation between the ranking in 2011 and rating change was 0.35, $p<0.01$, while in soccer it was 0.28, $p<0.05$.

*Testing* $H_9$: Heraclitus effect.

$H_9$ was tested by first completing a regression analysis with the ranks in 2011 as independent and the ratings in 2012 as dependent variables. The focus was now on the standardized residuals, which represent the unpredicted part of the ratings from 2012. According to the Heraclitus effect the residuals should be mainly positive for the very top positions (i.e. their performance is better than expected) and mainly negative for the more average positions (i.e. their performance is worse than expected). The results can be found in Table 2. The correlation for the top 25 is significantly different from the correlation for the other subsamples both for chess and for soccer (each $p$ was at least smaller than 0.001).

**Table 2:**
Stability of ranks and ratings for different subgroups between 2011 and 2012.

| Chess | | Soccer | |
|---|---|---|---|
| Subsample | Correlation | Subsample | Correlation |
| Rank 1-25 | -0.69** | Rank 1-25 | -0.22 |
| Rank 26-50 | 0.32 | Rank 26-50 | 0.66*** |
| Rank 51-75 | 0.24 | | |
| Rank 76-100 | 0.74*** | | |

Note: ** = $p<0.01$; *** = $p<0.001$

## Discussion

While it has been demonstrated that in most distributions under consideration the proposed effect of larger differences at extremal values holds (e.g. Simonton, 2000, 2009b), these distributions are mathematical idealizations and only approximate reality. While this approximation often works quite effectively, especially at extremal values it can sometimes deviate. In this contribution we gave a mathematical explanation (see $H_1$). Moreover and most importantly, we also demonstrated that the effect holds true for a range of natural distributions like normal distributions or Pareto distributions (see $H_2$).

In the next step we considered the dynamic of top positions. We showed mathematically that the reliability and particularly the stability are higher among the top positions compared to average positions (see $H_3$ and $H_4$).

In our study concerning single chess players and national soccer teams we demonstrated that performance was reliably measured in both sports. The stability of the ratings over one year was very high (see $H_5$). We were able to replicate the well-known result that the average differences of the performance of two successively ranked individuals or teams increase with better ranks (see $H_6$) (Simonton, 2000).

Subsequently three new empirical effects about the dynamics of top positions have been predicted and demonstrated to manifest in the data samples. The predicted stabilization effect is the empirical counterpart to $H_4$. Indeed, ranks and ratings turned out to be more stable among the top players (see $H_7$). It is therefore more likely, for example, that the winner of a gold medal at the Olympics could repeat her triumph than that the winner of the bronze medal could repeat her podium place.

The reversed Matthew effect (see $H_8$) adds dynamic aspects to these predominantly statistical findings. While the very best individuals contribute disproportionately to top achievements, as has been established in many studies (e.g. Shavinina, 2003; Simonton, 1997), their subsequent increases are lower than the increases of those who come after them in the rankings. Nevertheless, this makes sense as otherwise there would be an explosion of ever increasing performances. However, occupying a top position could also have been expected to come with many benefits. For example the best coaches are interested in training the best players (see also Ziegler & Baker, 2013). This dynamic manifests itself in the Heraclitus effect. It postulates that while those holding top positions have lower performance increases, they still are higher than predicted by a linear approximation. This could be demonstrated both for chess and soccer (see $H_9$).

Eminent persons and eminent teams accomplish a disproportionally high contribution rate to top achievements, which has been used to justify a particularly extensive investment into their education and promotion (compare Shavinina, 2003; Ziegler, 2008a, 2008b). Indeed, a lot of effort has been put into identifying, enabling and improving people or groups of people with the potential to accomplish high levels of achievement. Investigations into the smart fraction theory corroborate these notions empirically. This theory postulates that gifted and talented persons are especially important for societal, cultural, and scientific development. Rindermann et al. (2009) analysed data from the most important international comparative school studies in order to test these assumptions: TIMSS 1995-2007, PISA 2000-2006 and PIRLS 2001-2006. Their research encompassed 90 different countries. The upper level group comprised students with school achievements in the top 5% (equivalent to a within country IQ of at least 125). The studies compared these students' achievements with those of two other groups: students of average ability and students whose school achievements were below the 5th percentile. As it turned out, the performance of the smart fraction was closest connected to positive outcomes: wealth (GDP) of a nation, patent rates, Nobel Prizes, high technology exports, numbers of scientists, political variables (government effectiveness, rule of law, etc.), and cognitive development. The smart fraction was especially important for societal development.

Our results contribute two new facets to the discussion about investing in the advancement of eminence in groups and individuals. As top positions can be held for a longer time (see the stabilization effect), it seems that their successful education is especially worthwhile. Secondly, even eminent persons have the potential to increase their performance further. While this is more difficult than it is to increase the performance of average people (reversed Matthew effect), it still works better than statistically predicted (Heraclitus effect). Speaking in concrete terms, this means, for example, that while Usain Bolt could probably not increase his personal best as considerably as an athlete who is still just starting out, he might still have the potential to break his incredible world record of 9.58 seconds sooner than might be expected.

## Author note

Albert Ziegler jr. is a MALOA Fellow at the University of Leeds. Annika Heckel is a doctoral student and Scatcherd scholar at the University of Oxford. Albert Ziegler is Chair Professor for Educational Psychology, Excellence Research and Gifted Education at the University of Erlangen-Nuremberg.

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