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Integrating mathematical abilities and creativity in the assessment of mathematical giftedness

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Abstract

This study aims to examine the structure of the relationship between intelligence and mathematical giftedness and build a comprehensive model to describe this relationship and the nature of mathematical giftedness. This study also purports to clarify the structure of components of mathematical ability. The third objective is to examine whether students who were identified by two different instruments – (a) mathematical ability and creativity instrument and (b) intelligence instrument – have statistically significant differences across the components of mathematical ability. That is, we want to investigate if variance in identification may be explained by variance in mathematical abilities exhibited by these individuals. To achieve these goals, this study proposes a new domain-specific identification instrument for the assessment of mathematical giftedness, assessing mathematical abilities and creativity. The study was conducted among 359 4th, 5th and 6th grade elementary school students in Cyprus, using two instruments measuring mathematical ability and mathematical creativity and fluid intelligence. The results revealed that mathematical giftedness can be described in terms of mathematical ability and mathematical creativity. Moreover, the analysis illustrated that intelligence is a predictor of mathematical giftedness. Furthermore, the analysis revealed that different groups of students are identified by each type of testing; that is, through the mathematical instrument and the intelligence instrument. This variance may be explained by performance in specific categories of tasks.

Key words: giftedness, creativity, mathematical ability, intelligence

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Introduction

In recent years, intelligence testing as the exclusive means of identification of giftedness has received extensive criticism by a number of researchers (Dai, 2010; Lohman & Rocklin, 1995). Contemporary conceptualizations of giftedness acknowledge the multidimensionality (Gagné, 2003; Renzulli, 1978, 2002) and the domain specificity of the concept (Csikszentmihalyi, 2000; Clark, 2002). Hence, former identification processes measuring giftedness solely using intelligence instruments, should be enriched with other domain specific instruments measuring all dimensions of giftedness.

In the field of mathematical giftedness, identification was in many cases conducted through intelligence testing with subtests designed to assess mathematical giftedness, such as the Naglieri Non-Verbal Ability Test (Naglieri, 1997), the Wechsler Intelligence Scale for Children Matrix reasoning Test (Wechsler, 1999) and the Raven Progressive Matrices (Raven, Raven, & Court, 2003). These subtests focus on visual perception, spatial ability and the ability to distinguish patterns and find missing elements. At the same time, instruments designed to capture mathematical giftedness, such as TOMAGS (Ryser & Johnsen, 1998) include tasks that are aligned with curriculum standards in mathematics, hence measuring mathematical knowledge more than mathematical reasoning processes. In contrast, we would argue that identification should attempt to capture students' mathematical reasoning abilities, rather than mathematical knowledge, because reasoning skills distinguish gifted from non-gifted students in mathematics.

Given these facts, the concept of giftedness should be expanded, in order to encompass contemporary conceptions, models and approaches. Thus, mathematical giftedness may be expressed as a multidimensional construct that is domain specific. To this end, it is the purpose of this study to integrate mathematical abilities and creativity into the assessment of mathematical giftedness through a theoretical model and to translate it into an empirically examined identification process of mathematical giftedness. At the same time, we will show that there is a discrepancy in identifying mathematically gifted students with conventional intelligence tests and specifically designed mathematical instruments, thus suggesting a new way of identifying giftedness in mathematics.

The paper is organized as follows. First, the theoretical background is presented, addressing the identification of giftedness alongside with intelligence testing followed by a discussion on the construct of mathematical giftedness. Then, the study is presented, with attention to the design of a new identification process for giftedness in mathematics. Afterwards the construct validity of the proposed identification process is examined. This will be followed by a comparison of students who may be identified through a mathematical giftedness instrument in contrast with traditional measures of giftedness, such as intelligence tests, with the aim to explain the divergence in identification by two different instruments. Finally, the contribution and conclusions of the study are discussed.

Theoretical background

In this section, the topic of identification of giftedness is discussed, followed by a discussion on the construct of giftedness in mathematics.

Identification of giftedness

A systematic and scientific approach to the concept of giftedness began in the late 19th century, initially through the study of intelligence. Intelligence refers to individual differences in a set of cognitive abilities important for learning and problem solving, such as understanding complex ideas, engaging in various forms of reasoning, and effectively dealing with real life challenges (Neisser et al., 1996, cited in Dai, 2010). In the first half of the 20th century, Binet (1905, 1916) and Terman (1925) made distinctive efforts to create valid IQ (Intelligence Quotient) tests for measuring intelligence.

Terman shared the conviction that intelligence is a general human ability, in a large extent genetically determined and that it can be measured objectively with an intelligence test, such as that created by Binet and Simon. Terman (1925) defined giftedness as the possession of high mental power measurable by intelligence tests, suggesting the use of IQ scores above 140 as an identification criterion. Although this was one of the very first definitions of giftedness, the exclusive reliance on one score of an intelligence test, as a way of identifying gifted students, promoted an absolutist view of giftedness (Brown, Renzulli, Gubbins, Siegle, Zhang, & Chen, 2005).

The unidimensional definition of Terman was followed by other psychometric definitions that used a quantitative approach in viewing giftedness in terms of cut-off points on certain criteria, with IQ considered as the only or predominant index of giftedness. As a result, the field was dominated by narrow conceptualizations of giftedness (Hong & Milgram, 2008), inhibiting expansion of the conceptualization of giftedness for many years. Nevertheless, years later, the Procrustean bed notion of IQ tests was challenged (Dai, 2010). Indeed, several researchers realized the diversity of abilities of gifted individuals and used these to argue against measuring giftedness with single IQ instruments (Passow, 1981).

Further, studies by Hunt (1999, 2006) have shown that intelligence is more differentiated at the very high end of the spectrum. Explicitly, students with similar high IQ scores may differ in cognitive profiles and, hence, differ in cognitive strengths and weaknesses. Whilst these students may be thought as equally ‘gifted’, this identical IQ score may perhaps designate different things for each of them (Dai, 2010). In cases of domain specific giftedness, such as in mathematical giftedness, this finding is of particular concern. To illustrate, an identical IQ score for two students may conceal mathematical giftedness for one student and possibly artistic giftedness for the other.

Following these arguments, the objective for which an identification process is implemented, impacts on the use of identification instruments. In this direction, Milgram and Hong (2009) criticized the use of narrow measures used in schools such as IQ tests and school grades, since students whose potential differs from the abilities measured may be systematically excluded from gifted provisions. Moreover, it is of concern that these tests do not sufficiently identify high ability in mathematics. As Miller (1990) pinpointed, mathematical talent is a specific ability, whereas an IQ score is a summary of many different aptitudes and abilities, only some of which are related to mathematical ability.

Intelligence tests as a means to identify giftedness have been criticized for the information they provide. To be more specific, IQ tests solely provide information on how a student performed based on age norms, with no information provided on the process of obtaining the particular score (Dai, 2010). Moreover, intelligence testing as a selection process does not offer information on how provision should be differentiated to accommodate the needs of identified children (Lohman & Rocklin, 1995). Hence, both evaluation of academic performance and cognitive abilities should be used (Naglieri & Ford, 2003), despite their conceptual differences. Since the construct of giftedness itself is so complex, it is highly difficult for one test to measure all the behaviors that may be presented by gifted persons (Salvia & Ysseldyke, 2001). As a result, a combination of valid, reliable, sensitive and objective tools should be used in order to collect information for a student (Bicknell, 2009; Coleman, 2003; Davis & Rimm, 2004).

In contrast with theories that point to a single intelligence score, assuming that intelligence is a unidimensional entity, the experiential structuralism theory (Demetriou et al., 2002) provides an alternative view on intelligence by focusing on the structure of the mind and providing various reasoning abilities to describe it. According to this theory, the human mind involves a set of environment-oriented Specialized Structural Systems (SSSs), sets of specialized abilities with which a person can mentally manipulate and understand. Five SSSs have been proposed: (1) the qualitative-analytic, (2) the quantitative-relational, (3) the causal-experimental, (4) the spatial-imaginal, and (5) the verbal-propositional. This categorization of reasoning processes might be of particular importance in the field of intelligence and giftedness, since it distinguishes among various reasoning abilities rather than focusing on general intelligence. For example, a person may have developed each of these abilities to a different level. We may assume that gifted individuals may have developed all of these abilities to a great extent. Moreover, these abilities could be transferred to the field of mathematics in order to describe mathematical ability as a component of mathematical giftedness.

In recent years, multidimensional definitions for giftedness have been proposed expanding in several dimensions rather than just intelligence, thus integrating several factors to describe the concept. For example, Gagné (2003) proposed the differentiated model of giftedness and talent, a model that distinguishes between giftedness and talent. According to Gagné (2003), giftedness is the possession and use of untrained and spontaneously expressed natural abilities in five ability domains, such as the intellectual domain. Therefore, natural abilities, which are prerequisites for giftedness, according to Gagné, are more than intelligence. Talent denotes the exceptional mastery of systematically developed abilities or competencies in Gagné's terms, in at least one field of human activity. According to this model, the degree of these natural abilities needs to place the individual in the top 10% of age peers. An important element in this model is the developmental process, in which gifts are progressively transformed into talents, over a considerable time period and after the systematic pursuit by individuals towards a particular excellence goal. Taking into account this relationship between gifts and talents, we can conclude that a person can be gifted without necessarily being talented (as with the case of underachievers), but not vice versa. We believe that there are several elements in Gagné's model that could be incorporated in a model of mathematical giftedness. More specific, Gagné points to the difference between domain specific talent and intelligence, which is a natural ability prerequisite for giftedness. Natural abilities alone are not enough since a person's natural abilities should be developed into a talent, in our case mathematical giftedness. Thus, it is not sufficient to identify domain specific giftedness using intelligence tests, unless this is complemented by other domain specific instruments. In addition, we adopt Gagné's belief that in reference to natural abilities, these should be developed in that extent so that the gifted person is among the top 10% of its age peers.

To address the call for broader definitions beyond traditional notions of IQ (Lohman, 2009), Ziegler (2009) stressed the need for theories, concepts and definitions that are more domain specific, such as mathematics, and less trait oriented. In the light of this discussion, the following section comments on the notion of mathematical giftedness.

Giftedness in mathematics

In the domain of mathematics, various attempts have been made to identify the cognitive characteristics of mathematically gifted students (Greenes, 1981; House, 1987; Krutetskii, 1976; Miller, 1990; Osborne, 1981; Sowell, Zeigler, Bergwall, & Cartwright, 1990; Waxman, Robinson, & Mukhopadhyay, 1996). Our discussion will revolve around mathematical and creative abilities.

Mathematical ability. The review of the empirical research on the identification and portrayal of mathematically gifted students from the 1970s and 1980s has shown that gifted students differ in problem solving abilities from their average-ability peers (Sowell, Zeigler, Bergwall, & Cartwright, 1990). Similarly, studies by Zimmermann (cited in Wieczerkowski, Cropley, & Prado, 2000) concluded that mathematical giftedness entails "special ways of looking at and attempting to solve mathematical problems" (p. 415), as opposed to the acquisition of fixed knowledge.

Krutetskii's (1976) pioneering experimental work with school children, on the nature and structure of mathematical abilities is seminal, since most research studies on mathematically gifted students have drawn on this (Bicknell, 2009). According to Krutetskii (1976), mathematical giftedness is the "unique aggregate of mathematical abilities that open up the possibility of successful performance in mathematical activity" (p. 76). Among others, his research showed that capable pupils perceive the mathematical material of a problem both analytically and synthetically, generalize mathematical content rapidly and broadly with a minimal number of exercises, and show signs of flexibility of mental processes.

It was Krutetskii (1976) who proposed that mathematical giftedness is a special and unique qualitative combination of abilities. Thus, a mathematically gifted student may demonstrate only some of the traits and abilities described above. According to Sheffield (1994), some students may demonstrate several of these characteristics spontaneously, while others may reveal their abilities only when facing challenging mathematical situations. Problems that allow students to demonstrate their potential in mathematics should also demand the use of multiple reasoning methods. Greenes (1997) argues that the processes that students may use during problem solving include analogical, inductive, deductive, spatial, proportional and probabilistic reasoning. According to OECD (2010), in order to be mathematically literate, an individual should be able to "formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena…and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens" (p. 4).

The relationship between spatial abilities and mathematical giftedness has been a subject of investigation by a number of researchers. One of the conclusions of the literature review of empirical research on giftedness from the 70s and 80s was that mathematically gifted students differ in spatial abilities from their average-ability peers (Sowell, Zeigler, Bergwall, & Cartwright, 1990). Because of the overemphasis on analytical tasks at school, there is a possibility for spatially gifted students to underachieve (Diezmann & Watters, 1996). It is of particular concern that spatially gifted children may not be identified by current practices (Shea et al., 2001). Webb, Lubinski, and Benbow (2007) stressed that spatial ability is important for talent identification and should not be neglected. They claimed it is possible to identify a neglected number of mathematically gifted students, when focused on spatial ability. This perspective was supported by Shea and colleagues (2001) who also concluded that verbal and quantitative abilities alone, which are the most frequently assessed areas of intelligence, were insufficient descriptors of gifted children. Hence, the identification of spatially gifted students is only possible when they are provided with spatial tasks that allow them to make their ability evident.

Creative ability. Creativity has gained an important place within the context of gifted education (Kaufman, Plucker, & Russell, 2012), with a diversity of views on the relationship between giftedness and creativity. Several researchers argue that creativity is a specific type of giftedness (e.g., Sternberg, 1999, 2005), while other researchers consider creativity to be a critical part of giftedness (Renzulli, 1978, 1986), and yet others advocate that they are two autonomous features of individuals (Milgram & Hong, 2009).

Recent studies also make a distinction between general and specific creativity (Hong & Milgram 2010; Piirto, 2004; Simonton, 1999). Although Plucker and Zabelina (2009) report a lack of literature with regard to the concept of creativity in the field of mathematics education, one may trace a number of studies investigating the relationship between mathematical giftedness and creativity (e.g. Sriraman, 2005). According to Sriraman (2005), creative students exist in the "fringes" (p. 29) of the set of mathematically gifted students. In other words, mathematically gifted students are also mathematically creative, but at the same time, one may be creative but not mathematically gifted.

Recently, Kattou, Kontoyianni, Pitta-Pantazi, and Christou (2013) have also shown that mathematical creativity is a subcomponent of mathematical ability.

There is a number of studies attesting to the relationship between mathematical giftedness and creative behaviors. For example, Wolfle (1986) reported that mathematically gifted students develop unique solutions to common problems, whereas Greenes (1981) commented on their ability to interpret problem information in original ways. Miller (1990) observed gifted students' ability to work with mathematical problems in flexible and creative ways rather than in a stereotypic mode.

As suggested by the literature, mathematical creativity is an important dimension that should be incorporated into the identification of giftedness in mathematics. One of the sources of talent loss is the underestimation of the value of creative thinking and, thus, not emphasizing it in mathematics teaching and identification processes in schools (Milgram & Hong, 2009). In a recent review of the state of creativity assessment, Kaufman, Plucker and Russell (2012) conclude that, despite the many flaws present in every type of creativity measurement, creativity should be included as part of a gifted assessment battery.

Leikin (2009a) suggested a model for the assessment of creativity through the use of multiple solution mathematical tasks. Leikin and her colleagues thoroughly investigated multiple-solution connecting tasks, which they describe as "tasks that contain an explicit requirement for solving the problem in multiple ways" (Leikin & Levav-Waynberg, 2008, p. 234). Leikin (2009a) provided operational definitions of mathematical creativity and a scoring method for the assessment of creativity, based on fluency, flexibility, and originality, following Torrance (1974). There are also other researchers that used the concepts of fluency, flexibility and originality to define mathematical creativity (e.g. Gil, Ben-Zvi, & Apel, 2007). Gil, Ben-Zvi, and Apel (2007) described fluency as the ability to produce many ideas, flexibility as the number of approaches observed in a solution, and originality as the possibility of holding extraordinary, new and unique ideas. This definition of the three components of creativity was also used by Silver (1997). Therefore, in this way it is possible to quantitatively compare the mathematical creative performance for the same task (Pelczer & Rodríguez, 2011).

The differences observed in mathematically gifted students in mathematical and creative abilities when compared to non-gifted students taken together, affirm the need for a new identification process for mathematical giftedness (Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2011; Kontoyianni, Kattou, Pitta-Pantazi, & Christou, 2011). To this end, this study will propose an equitable identification process accompanied by a new identification instrument aiming to recognize mathematical giftedness in students aged 10 to 12 years.

Purpose of the study

Prior research in the field of giftedness focused on the examination of general giftedness rather than domain-specific giftedness (Leikin, 2009a). As a result of the focus in general giftedness, there is limited emphasis on theoretical models of mathematical giftedness as well as specially designed procedures and instruments for students' identification. Based on this discussion, the main objective of this study is to examine the structure of the relationship between intelligence and mathematical giftedness and build a comprehensive model to describe this relationship and also describe the nature of mathematical giftedness.

Taking into account previous research findings, in this study we will proceed to the development and administration of an identification process for identifying mathematically gifted students in elementary school. We consider that mathematical ability is not a one-dimensional general component and it comprises of specific abilities. Thus, our second objective is to clarify the structure of components and processes of mathematical ability, using the framework provided by Demetriou, Christou, Spanoudis and Platsidou (2002). To this end, two alternative models will be tested. The first model will present the factor of mathematical ability consisting of 29 variables, as were the mathematical problem solving tasks, whilst the second model will have the factor of mathematical ability formed by five distinct reasoning abilities, matching the five SSSs proposed by Demetriou, Christou, Spanoudis and Platsidou (2002).

The exclusive reliance on intelligence testing has been found to be problematic in the case of identifying mathematical giftedness (Dai, 2010). However, in our study, we do not neglect intelligence. Rather, we acknowledge the important role of natural abilities and we want to clarify this relationship, by using both mathematical and intelligence instruments into the identification process of mathematical giftedness. Thus, we want to see what happens if the identification process is based in one of the two instruments; in other words, if the identified students as mathematically gifted, coincide or differ. Furthermore, in order to validate the relationship between the two constructs, we will examine whether students identified by two different instruments: (a) a mathematical ability and creativity instrument and (b) an intelligence instrument, have statistically significant differences across the components of mathematical ability. That is, we want to investigate if variance in identification may be explained by variance in mathematical abilities developed by these individuals.

The following section will discuss the development of the identification instrument supported by a discussion of the chosen methodological approach for this study.

This study

Leikin (2011) addressed the problem of a number of studies on mathematical giftedness that put emphasis primarily on general psychological traits of individuals, whilst they do not investigate the thinking processes of gifted students in mathematics in accordance with contemporary theories of mathematics education. Numerous of the distinctive behaviors of gifted students in mathematics are apparent during problem solving (Niederer & Irwin, 2001). Mathematically gifted students may be identified based on their high levels of reasoning (Sheffield, 1999), given that teachers provide opportunities for them to demonstrate their thinking and cognitive processes, thus distinguishing them from students who are hard workers, but not mathematically gifted students.

To identify mathematical giftedness in this sense, aptitute tests can be used. Aptitude tests are designed to measure specific abilities that develop over time or the potential for future achievement in specific areas. In comparison with mathematics achievement tests, aptitude tests place less emphasis on computational skills and more emphasis on mathematical reasoning skills; hence, their results are more useful in identifying mathematical giftedness (Miller, 1990).

Development of the test

The development of the identification tool used in this study, adopted and combined some of the main elements of two theoretical frameworks: (a) Demetriou's experiential structuralism theory (Demetriou et al., 2002), and (b) Gagné's (2003) differentiated model of giftedness and talent.

According to the experiential structuralism theory (Demetriou et al., 2002), the human mind involves a set of environment-oriented Specialized Structural Systems (SSSs), sets of specialized abilities with which a person can mentally manipulate and understand. Specifically, according to this theory there are five SSSs: (1) the qualitative-analytic, (2) the quantitative-relational, (3) the causal-experimental, (4) the spatial-imaginal, and (5) the verbal-propositional.

The qualitative-analytic system is responsible for the representation and processing of similarity and difference relations (Demetriou et al., 2002). The quantitative-relational system focuses on abilities and skills of quantitative specification and representation (Kargopoulos & Demetriou, 1998). The causal-experimental system deals with cause and effect relations. The spatial-imaginal system focuses on abilities such as mental rotation, image integration, and image reconstruction. The verbal-propositional system involves formal relations between mental elements. For mathematical abilities we considered the five abilities described in SSSs.

From this theory, we adopted the five SSSs as components of mathematical ability, since these abilities have been also acknowledged by other researchers as indications of mathematical giftedness (e.g. Bicknell, 2008; Krutetskii, 1976). For instance, Bicknell (2008) refers to the ability to perceive and process qualitative and spatial relationships; the ability to perceive and generalize; the ability to reason analytically, deductively and inductively; the ability to abbreviate mathematical reasoning and to find rational, economical solutions, as some of the characteristics of mathematically gifted students.

Combined with Demetriou's theory of the human mind we used elements of Gagné's (2003) differentiated model of giftedness and talent. From this model, we adopted the distinction between natural abilities and talent. It is assumed that a subgroup of students with high natural abilities (top 10% of an age group) may transform them into mathematical talent. In our model, mathematical giftedness is viewed as synonym to mathematical talent as expressed in Gagné's model. This talent is demonstrated through students' above average mathematical abilities and mathematical creativity. The decision to express mathematical giftedness as a combination of mathematical ability and mathematical creativity is justified in the theoretical background of the study.

Through this study, our contribution is to show that the different types of reasoning described in the SSSs can be used to illustrate components of mathematical ability. Much effort was put into the design of our test, in order not to end up with a content specific test, aligned with curriculum standards. Since we want to assess ways of reasoning, by creating an ability test containing mathematical problems aligned with the reasoning methods of the five SSSs and also problems requiring creative thinking, we believe that we may capture giftedness in mathematics.

Mathematical ability and mathematical creativity test

To achieve the goals of this study, we proceeded first to the design of a mathematical ability and mathematical creativity test. For the design of the first part containing tasks assessing mathematical ability, we considered that mathematical ability is not a unidimensional entity, rather it is a multidimensional construct which consists of spatial conception, arithmetic and operations, proper use of logical methods, formulation of hypotheses concerning cause and effect, and the ability to think analogically (Bicknell, 2009; Krutetskii, 1976). These types of reasoning are aligned with the five SSSs documented according to the experiential structuralism theory (Demetriou et al., 2002). Therefore, the mathematical abilities test consisted of 29 mathematical items measuring five abilities, whose description follows.

The *quantitative ability* tasks required students to focus on quantitative properties, such as number sense, relations between numbers, mental calculations and pre-algebraic reasoning. An example of a quantitative task is presented in Figure 1.

Kiki wanted to find the sum of the numbers 1678 and 364 using her calculator. She accidentally pressed the buttons 1378 + 362. What can she do to correct her mistake? (Note: You must give an answer without doing any mathematical operations on paper.)

A. Add 300 and subtract 2.
B. Add 3 and subtract 2.
C. Subtract 300 and add 2.
D. Subtract 302.
E. Add 302.

Figure 1:
Example of a quantitative task from the mathematical abilities test.

Tasks focusing on *causal ability*, required from students to investigate cause/effect relations, make hypotheses, test these hypotheses and arrive to conclusions based on experimentation, as it is presented in Figure 2.

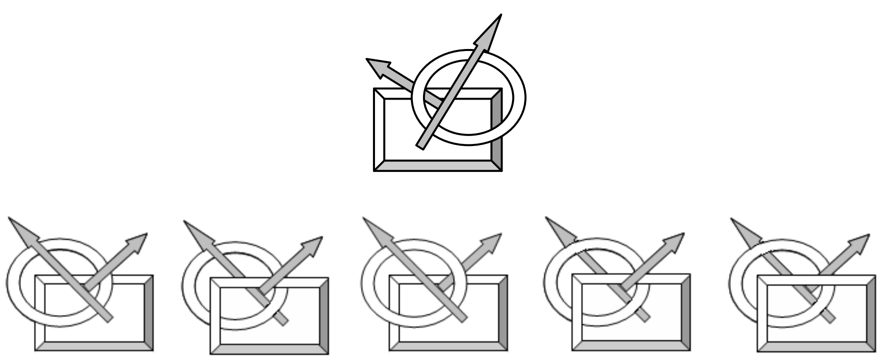
There are 20 yellow and 20 blue balls inside a big box. What can you do in order to increase the possibility of getting randomly a yellow ball?

A. Change the blue with red balls.
B. Put all the balls in a larger box.
C. Remove out of the box some blue balls.
D. Add some blue box balls.
E. Remove out of the box 7 blue and 7 yellow balls.

Figure 2:
Example of a causal task from the mathematical abilities test.

Third, there were problems assessing *spatial ability*. These problems, dealt with the notion of field-dependence, paper folding, perspective and spatial rotation (Figure 3).

Look carefully at the following figure. If you look at the figure from behind, what will you see?

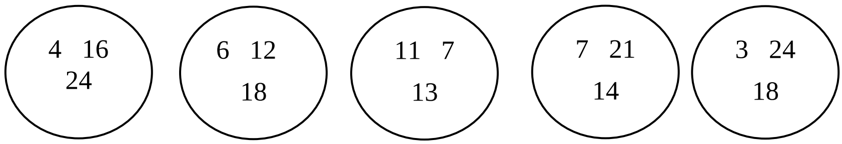


A. A B. B C. C D. D E. E

Figure 3:
Example of a spatial task from the mathematical abilities test.

The fourth category of tasks measured *qualitative ability*. These problems required students to focus on the representation and processing of similarity and difference relations based on written statements. Figure 4 shows a representative task of this category.

In each one of the following circles, there is a relationship between the numbers. If you want to place the number 32 in one of these circles, in which one of the five circles below will you place the number in order to fit?

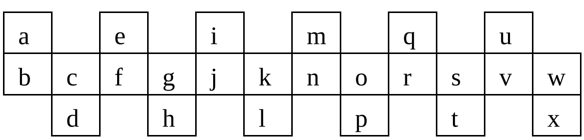


A. A B. B C. C D. D E. E

Figure 4:
Example of a qualitative task from the mathematical abilities test.

Lastly, our instrument included tasks assessing *inductive and deductive ability*. These were reasoning problems that demanded the use of inductive or deductive reasoning. An indicative task is presented in Figure 5.

The figure below is the key to find a code.



The code for the word "water" is 121, 11, 102, 31, 92.
What is the code for the word "oil"?

A. 28, 13, 33 B. 18, 15, 26
C. 81, 51, 62 D. 82, 31, 33
E. None of the above.

Figure 5:
Example of an inductive/deductive reasoning task from the mathematical abilities test.

The section of the test assessing mathematical creativity was designed upon the assumption that open-ended problems or multiple solutions tasks are appropriate for mathematical creativity, across the assessment of fluency, flexibility and originality of their solutions (e.g. Leikin 2007; Levav-Waynberg & Leikin 2009). Five open-ended multiple-solution mathematical tasks were used for this test and students were asked to provide: (a) multiple solutions; (b) solutions that were distinct from each other; and (c) solutions that none of his/her peers could provide. Here is an example of a creative task included in the test: “Start with number 7 and with the use of numbers and the arithmetic symbols (+,-, ×, ÷, ()) make 21 with as many ways as possible”.

Fluid intelligence instrument

For the measurement of fluid intelligence, we used the subtest Matrix Reasoning Scale from the Wechsler Abbreviated Scale of Intelligence (WASI) (Wechsler, 1999). Wechsler Intelligence Scales is one of the most widely accepted IQ tests for identifying gifted students (Silverman, 2009). The WASI Matrix Reasoning Scale includes 32 tasks for students of 9 to 11 years old and 35 tasks for students older than 11 years. There are four different types of tasks: pattern completion, classification, analogy and serial reasoning.

Participants

Three hundred and fifty nine elementary school students participated in the present study. Out of the 359 students, 143 students attended Grade 4, 118 students attended Grade 5 and 98 students attended Grade 6, in average public schools in Nicosia, in urban and suburban areas. The only requirement for a school to participate in the study was the existence of a computer lab. This requirement was due to the fact that the mathematical ability and mathematical creativity instrument was presented and solved in electronic form.

Tests and procedures

In order to accomplish the objectives of the study the two tests were administered to students: our test assessing mathematical abilities and mathematical creativity and the Matrix Reasoning Scale from the Wechsler Abbreviated Scale of Intelligence (WASI), to measure fluid intelligence.

Scoring and analysis

The items measuring mathematical abilities were marked as correct (1) or incorrect (0). Regarding the items measuring mathematical creativity, we assessed fluency, flexibility and originality of the solutions (Leikin, 2007). More specifically, we performed the following steps in order to assess a mathematical creative task: (a) Fluency score: we calculated the ratio number of the correct mathematical solutions that the student provided, to the maximum number of correct mathematical solutions provided by a student in the population under investigation. (b) Flexibility score: we calculated the ratio number of different types of correct solutions that the student provided, to the maximum number of different types of solutions provided by a student in the population under investigation. (c) Originality score: we calculated the frequency of each solution's appearance, in relation to the sample under investigation. A student was given the score 1 for originality if one or more of his/her answers appeared in less than 1% of the sample's answers. Correspondingly, a student was given a score of 0.8 if the frequency of one or more of his/her answers appeared in between 1% and 5%, 0.6 if the frequency of one or more of his/her answers appeared in between 6% and 10%, 0.4 if the frequency of one or more of his/her answers appeared in between 11% and 20%, 0.2 if one or more of his/her answers appeared in more than 20% of the sample's answers. Three different numbers (fluency, flexibility and originality scores) were calculated for each student, indicating the score in each mathematical creativity task. The total fluency, flexibility and originality scores were obtained by adding the respective scores across the five creativity tasks. The fluid intelligence test was assessed according to the manual of the Wechsler Abbreviated Scale of Intelligence (Wechsler, 1999).

Data analysis

Confirmatory factor analysis (CFA) in combination with descriptive statistics was employed for data analysis. In general, CFA is suitable to investigate whether a hypothesized structure (model) including cause-effect relationships between variables may represent a composite statistical hypothesis concerning patterns of statistical dependencies (Shirpley, 2000). Since the first aim of the study was to articulate and empirically test a theoretical model that addresses the relationship between mathematical giftedness and intelligence (Figures 6 and 7), CFA was applied in order to investigate the fit of the model to the data of the study. The statistical modeling program MPLUS (Muthén & Muthén, 1998) was used to test for model fitting.

In order to evaluate model fit, three fit indices are computed: the comparative fit index (CFI), the ratio of chi-square to its degree of freedom (x^2/df) and the root-mean-square error of approximation (RMSEA). According to Marcoulides and Schumacker (1996), for the model to be confirmed, the values for CFI should be higher than 0.90, the observed values for x^2/df should be less than 2 and the RMSEA values should be close to or lower than 0.08.

For the accomplishment of the second objective, that is the comparison among the participants that will be identified as gifted using each instrument (mathematical giftedness and intelligence test), crosstabs analysis was employed, using the statistical package SPSS. Multivariate analysis of variance was also conducted, with the quantitative, qualitative, causal, inductive/deductive reasoning, spatial and creative abilities used as dependent variables, in order to investigate differences between groups of students.

Results

Testing the structure of the proposed models

The first step in the analysis was to evaluate the construct validity of the mathematical giftedness test. Specifically we considered that mathematical giftedness is a combination of mathematical ability and mathematical creativity. Therefore, we wanted to examine whether the tasks used for the mathematical ability instrument may represent distinct types of abilities, as Demetriou, Christou, Spanoudis and Platsidou proposed (1992), or comprise a unified construct. The alternative models are presented in Figures 6 and 7. The model in Figure 6 assumes that mathematical ability is a unidimensional construct which in combination with mathematical creativity, through the assessment of fluency, flexibility and originality, constitute mathematical giftedness. In contrast, Figure 7 presents a model which assumes that mathematical ability consists of the qualitative-analytic, the quantitative-relational, the causal-experimental, the spatial-imaginal and the verbal-propositional abilities.

Figure 6 presents the structural equation model with the latent variables and their indicators. In particular, this model assumes that mathematical giftedness consists of mathematical abilities and mathematical creativity. The factor of mathematical ability was calculated as the sum of participants' performance in the 29 tasks of the mathematical abilities instrument. The analysis showed that the observed (students' performance on

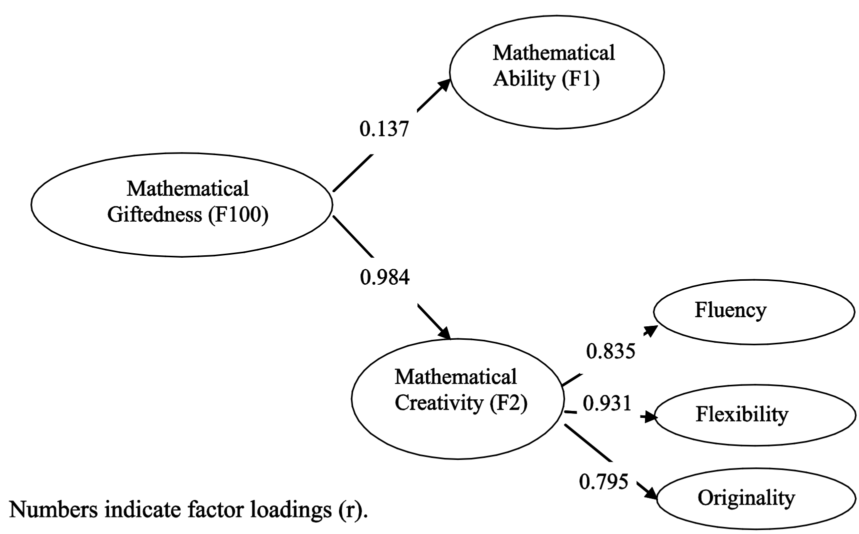


Figure 6:
The structure and loadings of the model considering mathematical ability as a one-dimensional construct.

tasks) and theoretical factor structures (the components of the theoretical model) did not match the data set of the present study and therefore they did not determine the "goodness of fit" of the factor model (CFI=0.901, x^2=75.00, df=3, x^2/df= 25.00, RMSEA=0.311), due to the fact that the index x^2/df is larger than 2 and the RMSEA index is not close 0.08.

Figure 7 presents an alternative model concerning the structure of mathematical giftedness. The difference from the first tested model is that in this case the factor of mathematical abilities is further analyzed into five factors describing five mathematical reasoning abilities. The analysis showed that the observed (students' performance on tasks) and theoretical factor structures (the components of the theoretical model) had a better match to the data set of the present study (CFI=0.990, x^2=29.26, df=19, x^2/df= 1.54, RMSEA=0.065), in comparison to the model previously presented in Figure 6. Thus, the analysis suggested that the model could represent distinct components necessary for the identification of mathematically gifted students.

Specifically, the statistically significant loadings of inductive/deductive reasoning abilities ($r=.725$, $p<.05$), quantitative abilities ($r=.667$, $p<.05$), qualitative abilities ($r=.625$, $p<.05$), causal abilities ($r=.475$, $p<.05$) and spatial abilities ($r=.306$, $p<.05$) showed that

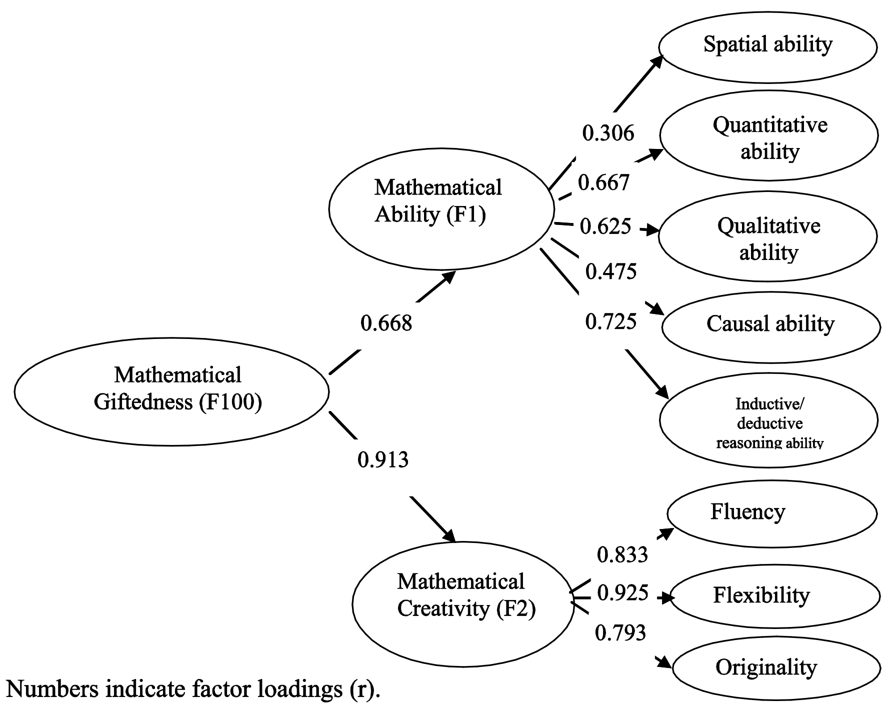


Figure 7:
The structure and loadings of the model considering mathematical ability as a multidimensional construct.

these abilities constitute mathematical abilities. Among the five mathematical abilities, inductive/deductive reasoning, quantitative and qualitative abilities contributed more to the construct, in contrast with causal and spatial abilities, which had the lowest loadings.

Likewise, the loadings for flexibility (r=.925 $p<.05$), fluency (r=.833, $p<.05$) and originality (r=.793, $p<.05$) comprised mathematical creativity. Both mathematical abilities (r=.668, $p<.05$) and mathematical creativity (r=.913, $p<.05$), constituted a higher order factor, that of mathematical giftedness. The loadings of these two factors suggested however that mathematical creativity contributed more in mathematical giftedness than mathematical abilities.

The second objective of the study was to examine the relationship between mathematical giftedness and intelligence. Due to this fact we considered that fluid intelligence may predict mathematical giftedness, as it is presented in Figure 8. The model (figure 8) fits the data of the study (CFI=0.985, x^2=41.955, df=25, x^2/df= 1.67, RMSEA=0.066). Specifically, the statistically significant loadings of inductive/deductive reasoning abilities (r=.714, $p<.05$), quantitative abilities (r=.672, $p<.05$), qualitative abilities (r=.633, $p<.05$), causal abilities (r=.458, $p<.05$) and spatial abilities (r=.329, $p<.05$) showed that these

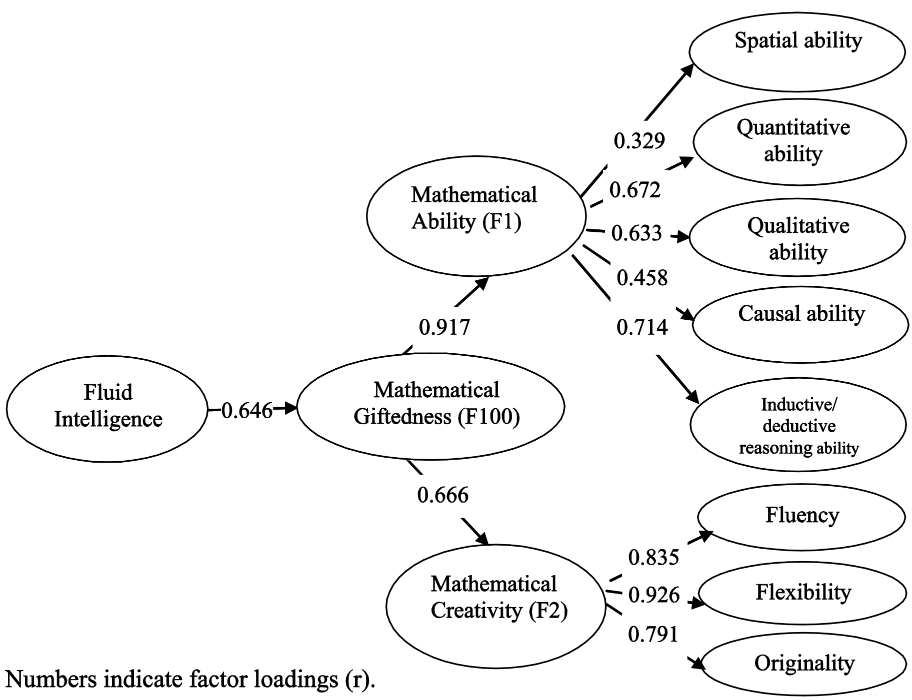


Figure 8:
The structure and loadings of the proposed model for the relationship among mathematical giftedness and intelligence.

abilities constitute mathematical abilities. Among mathematical abilities, inductive/ deductive reasoning, quantitative and qualitative abilities contributed more to the construct. Inductive and deductive reasoning is very important and most often emphasized in mathematics curricula, since it relates to students' ability to generalize and think abstractly. At the same time quantitative and qualitative reasoning is also highlighted in teaching. Students are often asked to find the similarities and differences amongst shapes, numbers and previously solved tasks and through these processes identify several relations. In contrast, spatial abilities had the lowest loadings on to mathematical ability, with causal ability having the second lowest loading. Likewise, the loadings for flexibility ($r=.926$, $p<.05$), fluency ($r=.835$, $p<.05$) and originality ($r=.791$, $p<.05$) comprised mathematical creativity. Between the three components of mathematical creativity, flexibility was shown to have the highest loading onto mathematical creativity, revealing the importance of providing different solutions to a problem. However, it needs to be noted that the loadings of fluency, flexibility and originality are high. Both mathematical abilities ($r=.917$, $p<.05$) and mathematical creativity ($r=.666$, $p<.05$), constitute a higher order factor, that of mathematical giftedness. Between mathematical ability and mathematical creativity, results showed that mathematical ability contributed more to the construct of mathematical giftedness. In addition, the analysis revealed that fluid intelligence could significantly predict mathematical giftedness ($r=.646$, $p<.05$). Thus, although they are different constructs, intelligence has a role to play in the identification of mathematical giftedness.

Our next step was to clarify the contribution that each instrument has to make into the identification of mathematical giftedness. More specific, we wanted to investigate whether there was a variation in individuals identified by each type of test.

Variation between the individuals identified by the two tests

Since mathematical giftedness is a composition of mathematical ability and mathematical creativity, we calculated a new score, by summing up the scores on the corresponding tests. Furthermore, it was our aim to examine whether the population that belongs to the highest 10% of the mathematical giftedness score is the same with the population that belongs to the highest 10% of the intelligence test score. Using crosstabs analysis, four groups of students were identified: (a) Group 1: Students who belonged in the highest 10% in both tests (hereafter we will call this group Gifted M-IQ), (b) Group 2: Students who belonged in the highest 10% of the intelligence score but not on the mathematical giftedness score (hereafter we will call this group Gifted IQ), (c) Group 3: Students who belonged in the highest 10% of the mathematical giftedness score but not on the intelligence score (hereafter we will call this group Gifted M), (d) Group 4: Students who did not belong in the highest 10% neither on the intelligence score nor on the mathematical giftedness score (hereafter we will call this group Non-gifted).

Concretely, 17 out of 359 students (Gifted M-IQ) were identified as gifted based both on their intelligence score and their mathematical giftedness score. Moreover, there were 19 students that were identified as mathematical gifted using the mathematical giftedness score but they were not identified using the intelligence test (Gifted M) and vice versa, 19 students that are not mathematically gifted were identified as gifted using the intelligence test (Gifted IQ). The remaining 304 participants were not identified as gifted with neither of the two tests (Non-gifted). The four groups of students are presented diagrammatically in Table 1.

Furthermore, we wanted to investigate the differences among the above mentioned groups of students, in order to examine which factors differentiate the results obtained by test scores. MANOVA took place, as it is presented in Table 2.

Comparing Gifted M-IQ with Gifted IQ students, statistically significant differences appear across participants' scores in quantitative ability (p=.001), qualitative ability (p=.001), causal ability (p=.002), inductive/deductive reasoning ability (p=.001) and creative ability (p=.008). However, the two groups behaved similarly in regard to spatial ability (p=.974). Given this finding, we may conclude that students belonging in these two Groups have similar score on spatial ability; an indicator that the intelligence test may mainly identify the spatially able students. We should remind that in our mathematical instrument, spatial reasoning was the factor with the lowest loading to mathematical ability. However, we should also bear in mind that in the context of the intelligence test used in this study, students were shown an array of pictures with one missing square, and then selected the picture that fitted the array from five options. Thus, since the intelligence subtest was mainly dependent on spatial ability it is of no surprise that students identified as gifted by both tests behaved similarly in spatial ability tasks.

Moreover, Gifted M and Gifted IQ students behaved similarly regarding spatial (p=.886) and quantitative abilities (p=.064), in contrast to qualitative (p=.031), causal (p=.009), inductive/deductive reasoning (p=.001) and creative (p=.007) abilities. We can assume that the intelligence test as a means to identify gifted mathematicians may select only the spatial and quantitative abilities and not the other abilities that are considered as components of mathematical giftedness. From these findings, we may assume that in former years, using traditional mathematical and intelligence tests, it was enough for someone to be considered mathematically gifted if this person had a good spatial sense and also was good in calculations. Nowadays, this is not enough. As our results show, there are other mathematical reasoning abilities equally important, such as qualitative, causal, inductive/deductive and creative reasoning abilities.

As we can see from Table 2, Non Gifted students had statistically significant differences across the five mathematical abilities (spatial: p=.018, quantitative: p=.001, qualitative: p=.001, causal: p=.001, inductive/deductive reasoning: p=.001) and creativity (p=.001) with Gifted M-IQ students. The same results as the previous ones were obtained when comparing Non Gifted with Gifted M (spatial: p=.001, quantitative: p=.001, qualitative: p=.001, causal: p=.001, inductive/deductive reasoning: p=.001, creativity: p=.001). On the contrary, non gifted students behave similarly with Gifted IQ students across causal (p=1.000), inductive/deductive reasoning (p=.419) and creative ability (p=.193). Therefore, students identified as mathematically gifted with the use of an intelligence test have several similar cognitive characteristics with non gifted students. Due to this fact, a number of "gifted mathematicians" that have been assessed solely with intelligence instruments are not really gifted.

Table 1:
Groups of students according to their performance on the intelligence test and the mathematical giftedness test.

	High IQ	**Non High IQ**	**Total**
High Mathematics Ability	17 students	19 students	36 students
Non High Mathematics Ability	19 students	304 students	323 students
Total	36 students	323 students	359 students

Table 2:
Comparing the performance of the four groups of students on the intelligence test and the mathematical giftedness test.

		Gifted IQ	**Gifted M**	**Non Gifted**
Gifted M-IQ	Spatial	.974	.668	.018*
	Quantitative	.000*	.252	.000*
	Qualitative	.000*	.404	.000*
	Causal	.002*	.961	.000*
	Inductive/Deductive	.000*	.997	.000*
	Creative	.008*	1.000	.000*
Gifted IQ	Spatial	-	.886	.001*
	Quantitative	-	.064	.005*
	Qualitative	-	.031*	.004*
	Causal	-	.009*	1.000
	Inductive/Deductive	-	.001*	.419
	Creative	-	.007*	.193
Gifted M	Spatial	-	-	.000*
	Quantitative	-	-	.000*
	Qualitative	-	-	.000*
	Causal	-	-	.000*
	Inductive/Deductive	-	-	.000*
	Creative	-	-	.000*

* Statistical significant differences, $p<.05$.

Discussion

In the past decades, the field of giftedness was dominated by intelligence theories and thus identification methods focused solely or primarily on intelligence scores. In recent years, it has been acknowledged that giftedness is not a unidimensional construct and as such, a variety of information and behaviors should be investigated during identification (Salvia & Ysseldyke, 2001). In addition, giftedness is domain specific (Csikszentmihalyi, 2000; Clark, 2002), and thus, domain- specific measures should be taken into account for identification purposes. In our case, in the field of mathematical giftedness, mathematical abilities and mathematical creativity have been discussed as components of mathematical giftedness (e.g. Renzulli, 1976).

Hence, one of the aims of this study was to articulate and empirically test two alternative theoretical models, clarifying the structure and components of mathematical ability; in the first model mathematical ability was composed of the 29 mathematical tasks, whilst in the second model mathematical ability was described across five components, as suggested by the SSSs described in Demetriou's experiential structuralism theory (2002). The construct of mathematical giftedness was defined across mathematical ability and mathematical creativity. The model that best described the construct of mathematical giftedness was the one where mathematical ability was not considered as a one-dimensional construct, but instead it was further analyzed into five distinct types of mathematical reasoning abilities. Hence, spatial, quantitative, qualitative, causal, and inductive/deductive abilities, were abilities exhibited by mathematically gifted children. This result is in accord with the results of House (1987), who among others, suggested the following traits as signals of mathematical giftedness: "early curiosity and understanding about the quantitative aspects of things, ability to think logically and symbolically about qualitative and spatial relationships; ability to perceive and generalize about mathematical patterns, structures, relations, and operations; ability to reason analytically, deductively, and inductively" (p. 9).

In particular, inductive/deductive, quantitative, and qualitative abilities, contributed more to mathematical ability. Inductive and deductive reasoning is often emphasized in mathematics curricula, since it is related to more general and abstract reasoning. Quantitative and qualitative reasoning is also stressed in mathematics classes. In contrast, spatial abilities have the lowest loadings on to mathematical ability while causal ability had the second lowest loading. We may explain this finding, by commenting that these two types of abilities are not emphasised in Cypriot classes, mainly because causal reasoning, where students are asked to form hypothesis, experiment, arrive to conclusions and judge statements is not often requested.

With respect to creativity, the loadings for flexibility, fluency and originality comprise mathematical creativity. Between the three components of mathematical creativity, flexibility was shown to have the highest loading onto mathematical creativity, revealing the importance of providing different solutions to a problem. Between mathematical ability and mathematical creativity, results show that mathematical ability contributes more to the construct of mathematical giftedness.

Furthermore, this study also aimed to examine the structure of the relationship between intelligence and mathematical giftedness. Intelligence as a natural ability is by no means underestimated in this study. Rather, its importance was acknowledged and as such, an intelligence test was incorporated and used complimentarily to a mathematical test in the identification process of mathematical giftedness. Our results revealed that intelligence is a strong predictor of mathematical giftedness. Given this result, intelligence should not be solely used for identification of mathematical giftedness rather this score is an indicator of potential giftedness in a domain.

The third aim of the study was to compare the characteristics among gifted students who have been identified using our mathematical giftedness instrument and the students who have been identified using an intelligence test. Therefore, it was our aim to examine whether the population that belongs to the highest 10% of the mathematical giftedness score is the same with the population that belongs to the highest 10% of the intelligence test score. Our findings confirmed that although there were a number of students identified as gifted with both intelligence and the mathematical giftedness scores, still a number of students would not have been identified with the sole use of the intelligence test whereas some high IQ students would have not been identified with the sole use of our mathematics test. That is, there is also a number of students that were not mathematically gifted but they were identified as gifted using the intelligence test. Subsequently, relying exclusively on an intelligence test for the identification of mathematical giftedness may result to misidentified mathematically gifted students or gifted students in another domain, other than mathematics.

Furthermore, we investigated the differences among the above mentioned groups of students, in order to examine which aspects were measured with each identification tool and what were the differences between them. Based on the data analysis, the intelligence test mainly identified students with high spatial and quantitative abilities. Traditionally, these two abilities were considered as a sign of high mathematical abilities and in fact, a number of intelligence scales were designed in a form of matrix reasoning, measuring visual processing, spatial perception and perceptual reasoning, such as the Naglieri Non-Verbal Ability Test (Naglieri, 1997), the Wechsler Intelligence Scale for Children Matrix reasoning Test (Wechsler, 1999) and the Raven Progressive Matrices (Raven, Raven, & Court, 2003). To succeed in this type of intelligence test a person should have a strong sense of spatial relationships, be able to work with patterns, such as finding missing elements in patterns or continuing patterns. Therefore, when we use intelligence scales to identify mathematically gifted students we will identify only the students having quantitative and spatial abilities developed, and neglect other abilities, such as the abilities to observe similarities and differences, to solve problems using inductive and deductive reasoning and to understand cause and effect relationships.

Summing up, our research has shown that intelligence testing may predict mathematical giftedness. Still, when used on its own it provided different results to that of our domain specific mathematics test. Thus, we would argue that the two instruments should be used complementary to each other in the identification process. The findings of this study also suggest that mathematical giftedness should not be perceived as a unitary construct. Rather, it should be perceived as a combination of mathematical creativity and mathematical abilities; namely quantitative, qualitative, spatial, causal/experimental and inductive/deductive abilities. This finding is in line with OECD's (2010) definition of mathematical literacy, according to which the mathematical literate person should possess mathematical reasoning, use mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena, as well as make well-founded judgments and decisions.

In the future, subsequent research efforts could investigate the impact of each of these abilities on the identification of mathematical giftedness. In addition, it would be of particular importance to investigate the impact on giftedness, when we invest in the development of the components of mathematical abilities as shown in this study during teaching.

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References

Bicknell, B. (2008). Gifted students and the role of mathematics competitions. *Australian Primary Mathematics Classroom, 13*(4). 16-20. Retrieved from http://www.aamt.edu.au/Publications-and-statements/Journals/Journals-Index/Australian-Primary-Mathematics-Classroom/APMC-13-4-16/(language)/eng-AU

Bicknell, B. A. (2009). *Multiple perspectives on the education of mathematically gifted and talented students.* (Doctoral Thesis, Massey University, Palmerston North, New Zealand). Retrieved from http://mro.massey.ac.nz/bitstream/handle/10179/890/02whole.pdf?sequence=1

Binet, A., & Simon, T. (1905, 1916). New methods for the diagnosis of the intellectual level of subnormals. In H.H. Goddard (Ed.), *Development of intelligence in children (the Binet-Simon-Scale)* (E.S. Kite, Trans., pp. 37±90). Baltimore: Williams & Wilkins.

Brown, S. W., Renzulli, J. S., Gubbins, E. J., Siegle, D., Zhang, W., & Chen, C. (2005). Assumptions underlying the identification of gifted and talented students. *Gifted Child Quarterly, 49,* 68–79. doi: 10.1177/001698620504900107

Clark, B. (2002). *Growing up gifted.* Merrill Prentice Hall, Upper Sadle River, NJ.

Coleman, M. R. (2003). *The identification of students who are gifted.* (ERIC Digest ED480431). Arlington, VA: ERIC Clearinghouse on Disabilities and Gifted Education. Retrieved from http://www.eric.ed.gov/PDFS/ED480431.pdf

Csikszentmihalyi, M. (2000). *Beyond boredom and anxiety.* San Francisco: Jossey-Bass.

Dai, D. (2010). *The nature and nurture of giftedness: A new framework for understanding gifted education.* New York, NY: Teachers College Press.

Davis, G. A., & Rimm, S. B. (2004). *Education of the gifted and talented* (5th ed.). Boston, MA: Allyn & Bacon.

Demetriou, A. (2002). Tracing psychology's invisible giant and its visible guards. In R. J. Sternberg & E. L. Grigorenko (Eds.), *The general factor of intelligence: How general is it?* (pp. 3–18). Mahwah, NJ: Erlbaum.

Demetriou, A., Christou, C., Spanoudis, G., & Platsidou, M. (2002). The development of mental processing: Efficiency, working memory and thinking. *Monographs of the society for research in child development, 67* (1, Serial No. 268). Retrieved from http://www.wiley.com/bw/journal.asp?ref=0037-976x

Diezmann, C. M. & Watters, J. J. (1996). Two faces of mathematical giftedness. *Teaching Mathematics, 21*(2), 22-25.

Friedman, R. C., & Shore, B. M. (eds.) (2000). *Talents Unfolding: Cognition and Development*. American Psychological Association, Washington, DC.

Friedman-Nima, R., O'Brien, B., & Frey, B. B. (2005). Examining our foundations: Implications for gifted education research. *Roeper Review, 28*(1), 45-52. doi:10.1080/02783190509554336

Gagné, F. (2003). Transforming gifts into talents: The DMGT as a developmental theory. In N. Colangelo & G. A. Davis (Eds.), *Handbook of gifted education* (3rd ed.) (pp. 60-74). Boston: Allyn & Bacon.

Gil, E., Ben-Zvi, D., & Apel, N. (2007).What is hidden beyond the data? Helping young students to reason and argue about some wider universe. In D. Pratt & J. Ainley (Eds.), *Proceedings of the Fifth International Research Forum on Statistical Reasoning, Thinking and Literacy: Reasoning about Statistical Inference: Innovative Ways of Connecting Chance and Data* (pp. 1-26). UK: University of Warwick. Retrieved from http://srtl.stat.auckland.ac.nz/srtl5/presentations

Greenes, C. (1981). Identifying the gifted student in mathematics. *Arithmetic Teacher, 28,* 14-18.

Greenes, C. (1997). Honing the abilities of the mathematically promising. *Mathematics Teacher, 90*(7), 582- 586.

Hong, E. & Milgram, R. M. (2008). *Preventing talent loss.* New York, NY.: Routledge.

House, P. A. (Ed.) (1987). *Providing opportunities for the mathematically gifted, K-12.* Reston, VA.

Hunt, E. (1999). Intelligence and human resources: Past, present and future. In P. L. Ackerman, P.C. Kyllonen & R.D. Roberts (Eds.), *Learning and individual differences: Process, trait, and content determinants* (pp. 3-28). Washington, DC: American Psychological Association.

Hunt, E. (2006). Expertise, talent, and social encouragement. In K. A. Ericsson, N., Charness, P. J. Feltovich & R. R. Hoffman (Eds.), *The Cambridge handbook of expertise and expert performance* (pp. 31–38). Cambridge, MA: Cambridge University.

Kargopoulos, P., & Demetriou, A. (1998). Logical and psychological partitioning of mind. Depicting the same map? *New Ideas in Psychology, 16,* 61–87. doi: 10.1016/S0732-118X(96)10021-0

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM: The International Journal on Mathematics Education, 45*(2), 167-181. doi: 10.1007/s11858-012-0467-1.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2011). On the comparison between mathematically gifted and non-gifted students' creative ability. *Paper presented at the 19th Biennial World Conference of the WCGTC*. Prague, Czech Republic.

Kaufman, J.C., Plucker, J.A., & Russell, C.M. (2012). Identifying and Assessing Creativity as a Component of Giftedness. *Journal of Psychoeducational Assessment, 30*(1), 60-73.doi:10.1177/0734282911428196

Kontoyianni, K., Kattou, M., Pitta-Pantazi, D., & Christou, C. (2011). Entering the world of mathematically gifted. *Paper presented at the 19th Biennial World Conference of the WCGTC*. Prague, Czech Republic.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren.* (J. Kilpatrick & I. Wirszup, Eds.) (J. Teller, Trans.). Chicago: University of Chicago Press. (Original work published 1968)

Leikin, R. (2009). Bridging research and theory in mathematics education with research and theory in creativity and giftedness. In R. Leikin, A. Berman & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 385-411). Rotterdam, the Netherlands: Sense Publishers.

Leikin, R. (2011). The education of mathematically gifted students: Some complexities and questions. *The Montana Mathematics Enthusiast, 8*(1&2), 167-188. Retrieved from http://www.math.umt.edu/TMME/vol8no1and2/index.html

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. In D. Pitta-Pantazi, & G. Philippou (Eds.), *Proceedings of the fifth conference of the European Society for Research in Mathematics Education – CERME-5* (pp. 2330–2339). http://ermeweb.free.fr/Cerme5.pdf. Accessed 17 Sep 2012.

Leikin, R., & Levav-Waynberg, A. (2008). Solution Spaces of Multiple-Solution Connecting Tasks as a Mirror of the Development of Mathematics Teachers' Knowledge. *Canadian Journal of Science, Mathematics and Technology Education, 8*(3), 233-251. doi:10.1080/14926150802304464

Levav-Waynberg, A. & Leikin R. (2009). Multiple solutions to a problem: A tool for assessment of mathematical thinking in geometry. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Conference of European Research in Mathematics Education* (pp. 776–785). Lyon, France: Institut National de Recherche Pédagogique. Retrieved from http://www.inrp.fr/editions/editions-electroniques/cerme6/

Lohman, D. F., & Rocklin, T. (1995). Current and recurrent issues in the assessment of intelligence and personality. In D. H. Saklofske & M. Zeidner (Eds.), *International handbook of personality and intelligence* (pp. 447–474). New York: Plenum.

Lohman, D. F. (2009). Identifying academically talented students: Some general principles, two specific procedures. In L. V. Shavinina (Ed.), *International handbook on giftedness* (pp. 971- 998). Amsterdam: Springer Science and Business Media.

Marcoulides, G. A., & Schumacker, R. E. (1996). *Advanced Capacity equation modelling: Issues and techniques*. NJ: Lawrence Erlbaum Associates.

Milgram, R & Hong, E. (2009). Talent loss in mathematics: Causes and solutions. In R. Leikin, A. Berman & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 149–163). Rotterdam, the Netherlands: Sense.

Miller, R. (1990). *Discovering Mathematics Talent.* ERIC Digest #E482.

Muthén, L. K., & Muthén, B. O. (1998). *Mplus user's guide*. Los Angeles, CA: Muthén & Muthén.

Naglieri, J. & Ford, D. (2003). Addressing underrepresentation of gifted minority children using the Naglieri Nonverbal Ability Test (NNAT). *Gifted Child Quarterly 47*(2), 155-160. doi: 10.1177/001698620304700206

Naglieri, J. A. (1997). *Naglieri Nonverbal Ability Test: Multilevel technical manual.* San Antonio, TX: Harcourt Brace.

Niederer, K. & Irwin, K.C. (2001). Using problem solving to identify mathematically gifted children. In M. Van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 431-438). Utrecht: The Netherlands.

OECD (2010). *Pisa 2012 Mathematics Framework: Draft subject to possible revision after the field trial.* Retrieved from http://www.oecd.org/pisa/pisaproducts/46961598.pdf

Osborne, A. (1981). Needed research: Mathematics for the talented. *Arithmetic Teacher, 28*(6) 24-25.

Passow, A. H. (1981). The nature of giftedness and talent. *Gifted Child Quarterly, 25,* 5–10.

Pelczer, I., & Rodríguez, F. G. (2011). Creativity assessment in school settings through problem posing tasks. *The Montana Mathematics Enthusiast, 8*(1&2), 383-398. Retrieved from http://www.math.umt.edu/tmme/

Piirto, J. (2004). *Understanding Creativity.* Scottsdale, AZ: Great Potential Press.

Plucker, J., & Zabelina, D. (2009). Creativity and interdisciplinarity: One creativity or many creativities? *ZDM: The International Journal on Mathematics Education, 41,* 5–11. doi: 10.1007/s11858-008-0155-3

Raven, J., Raven, J.C., & Court, J.H. (2003). *Manual for Raven's Progressive Matrices and Vocabulary Scales. Section 1: General Overview.* San Antonio, TX: Harcourt Assessment.

Renzulli, J. S. (1978). What Makes Giftedness? Reexamining a Definition. *Phi Delta Kappan, 60*(3), 180-184. Retrieved from http://www.pdkintl.org/kappan/index.htm

Renzulli, J. S. (1986). The three-ring conception of giftedness: A developmental model for creative productivity. In R. J. Sternberg & J. E. Davidson (Eds.), *Conceptions of giftedness* (pp. 53-92). New York, NY: Cambridge University Press.

Renzulli, J. S. (2002). Expanding the Conception of Giftedness to Include Co-Cognitive Traits and to Promote Social Capital. *Phi Delta Kappan, 84*(1), 33-58.

Salvia, J. & Ysseldyke, J.E. (2001). *Assessment*. (8th ed.). Boston, MA: Houghton-Mifflin.

Shea, D. L., Lubinski, D., & Benbow, C. P. (2001). Importance of assessing spatial ability in intellectually talented young adolescents: A 20-year longitudinal study. *Journal of Educational Psychology, 93*(3), 604-614. doi:10.1037/0022-0663.93.3.604

Sheffield, L. J. (1994). *The development of gifted and talented mathematics students and National Council of Teachers of Mathematics standards*. Storrs, CT: The National Research Center on the Gifted and Talented.

Sheffield, L. J. (1999). The development of mathematically promising students in the United States. *Mathematics in School, 28*(3) 15-18. doi: 10.2307/30212002

Shipley, B. (2000). A new inferential test for path models based on directed acyclic graphs. *Structural Equation Modeling, 7*(2), 206–218. doi: 10.1207/S15328007SEM0702_4

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 29*(3), 75-80. doi:10.1007/s11858-997-0003-x

Silverman, L. K. (2009). The measurement of giftedness. In L. V. Shavinina (Ed.), *International handbook on giftedness* (pp. 947-970). Amsterdam: Springer Science and Business Media.

Simonton, D. K. (1999). *Origins of genius: Darwinian perspectives on creativity.* New York: Oxford University Press.

Sowell, E. J., Zeigler, A. J., Bergwall, L., & Cartwright, R. M. (1990). Identification and description of mathematically gifted students: A review of empirical research. *Gifted Child Quarterly, 34*, 147–154. doi: 10.1177/001698629003400404

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education, 17,* 20–36. doi: 10.4219/jsge-2005-389

Sternberg, R. J. (1999). Intelligence as Developing Expertise. *Contemporary Educational Psychology, 24*(4), 359–375. doi:10.1006/ceps.1998.0998

Terman, L. M. (1925). *Genetic studies of genius: Vol. 1. Mental and physical traits of a thousand gifted children.* Stanford, CA: Stanford University Press.

Ryser, G. R. & Johnsen, S. K. (1998). *Test of Mathematical Abilities for Gifted Students (TOMAGS).* Austin, TX: Pro-ed.

Waxman, B., Robinson, N. M., & Mukhopadhyay, S. (1996).*Teachers nurturing math talented young children.* Storrs, CT: The National Research Center on the Gifted and Talented.

Webb, R. M., Lubinski, D., Benbow, C. P. (2007). Spatial ability: A neglected dimension in talent searches for intellectually precocious youth. *Journal of Educational Psychology, 99,* 397–420. Retrieved from http://psycnet.apa.org/?&fa=main.doiLanding &doi=10.1037/0022-0663.99.2.397

Wechsler, D. (1999). *Wechsler Abbreviated Scale of Intelligence.* San Antonio, TX: Psychological Corporation.

Wieczerkowski, W., Cropley, A. J., & Prado, T. M. (2000). Nurturing Talents/Gifts in Mathematics. In K. A. Heller, F. J. Mönks, R. J. Sternberg, & R. F. Subotnik (Eds.), *International handbook of giftedness and talent* (2 ed., pp. 413–425). Oxford, England: Elsevier Science.

Wolfle, J. A. (1986). Enriching the mathematics for middle school gifted students. *Roeper Review, 9,* 81-85. doi: 10.1080/02783198609553015

Ziegler, A. (2009). Research on giftedness in the 21st century. In L. V. Shavinina (Ed.), *International handbook on giftedness* (pp.1509-1524). Amsterdam: Springer Science and Business Media.