

# How brain research can contribute to the evaluation of mathematical giftedness

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## **Abstract**

In this paper we suggest that instruments of neuro-cognitive research enable the evaluation of giftedness in mathematics. We start with a literature review on the related topics presented so as to situate our suggestions within the existing research on giftedness and excellence in mathematics. This literature review allows us later to discuss our findings, which are based on neurocognitive data collected in a large-scale multidimensional examination of mathematical giftedness. Sampling procedure in the study was performed based on two orthogonal (in our view) characteristics: general giftedness (G) and excellence in mathematics (EM). In this paper we present findings that lead to a definition of the mathematically gifted population. We present selected results to provide evidence for our findings. In this paper we demonstrate three major findings:

A. Effects of G and EM factors are task-dependent both in behavioral and neurophysiological measures: the EM factor has significant main effects on tasks that require implementation of knowledge familiar to students from school mathematics. By contrast, the G factor has a significant main effect on insight-based problems which are not part of the school mathematical curriculum and, thus, require original mathematical reasoning.

B. Mathematical performance in gifted students who excel in mathematics (G-EM students) on insight-based tasks has specific characteristics in both behavioral and electrophysiological results.

C. G-EM participants exhibited superior performance in all the tests, showing a constant neuro-efficiency effect.

Based on these observations we suggest that mathematically gifted students are those who are both generally gifted and excel in mathematics.

**Key words:** Giftedness, Excellence in mathematics, Neurocognition, Evaluation, Problem solving

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## Introduction

Exceptional performance in mathematics or mathematical giftedness appears to be a topic of great interest to researchers and educators. In the scientific and pedagogical literature, there is wide and multilateral discussion on the subject of mathematical giftedness, its nature and main characteristics, educational challenges and perspectives associated with this phenomenon, and last (but not least) on the principles and methods of identification and assessment of mathematical giftedness. Even so, evaluation of individuals who are presented as overperformers or excelling in the field of mathematics is not an easy matter due to the lack of strong definitions of the phenomenon of mathematical giftedness. Consequently, development of tools for evaluation of individual abilities (especially high abilities) in the field of mathematics is not sufficient. Applying brain research to the study of mathematical giftedness seems to be of timely importance and can lead to an operative definition of mathematical giftedness and consequently to the development of tools that enable researchers to identify mathematical giftedness.

## 1. Background

### 1.1 Giftedness and excellence in mathematics

Mathematical giftedness is an extremely complex construct which implies high mathematical abilities. The construct of mathematical giftedness bridges two fields of educational psychology: gifted education and mathematics education. In the field of gifted education, mathematical giftedness is usually considered to be a distinct type of specific giftedness which is opposed to general giftedness (Piirto, 1999). General giftedness is usually measured by means of IQ tests, whereas mathematical giftedness implies high achievement in mathematics demonstrating strong mathematical skills. Mathematical giftedness among research mathematicians is expressed in the generation of original mathematical ideas and proofs that lead to advancements in mathematical theory. This leads to the question of how mathematical giftedness can be identified in school children. This paper examines brain activity associated with mathematical problem solving in 10<sup>th</sup> and 11<sup>th</sup> grade students.

#### *1.1.1 General giftedness, definitions, perspectives*

In the scientific literature, there is no universal definition of giftedness (e.g., Davis & Rimm, 2004; Ziegler & Raul, 2000); furthermore, there is ambiguity in the distinction between the concepts of giftedness and talent (Gagné, 1985) as well as between general giftedness and specific giftedness (Swanson, 2006). The psychometric definition of giftedness determined by Terman is an IQ (Intelligence Quotient) that is two SDs (standard deviations) above the population mean (usually 130) (Feldman, 2003; Silverman, 2009; Winner, 2000).

Over the years, the popular notion of "unitary" intelligence has been consistently challenged, as theorists started to broaden the notion of intelligence (Callahan, 2000). A num-

ber of theorists have developed broad, multi-dimensional formulations of giftedness and talent that are now widely accepted (Gardner, 1983/2003; Marland, 1972; Sternberg, 2000) and consider giftedness to be the result of the complex interaction of cognitive, personal-social, and socio-cultural influences. Note that all theories defining giftedness embody in their definition high academic abilities combined with some other unique qualities.

### *1.1.2 Mathematical abilities and skills*

The precise acquisition of mathematical abilities involves a broad range of general cognitive skills, including spatial perception, visuo-spatial ability, visual perception, visuo-motor perception, attention, and memory including working memory (Berg, 2008; Bull & Scerif, 2001; Butterworth, 1999; Geary, 1993; Hoard, Geary, Byrd-Craven, & Nugent, 2008; Meyer, Salimpoor, Geary, & Menon, 2010; Taub, Floyd, Keith, & McGrew, 2008). Together, these skills enable the acquisition, understanding, and performance of various mathematical activities (Ardila & Rosselli, 2002). High mathematical ability is often related to the students' problem-solving performance on complex tasks (Leikin, Koichu, & Berman, 2009, Davidson & Sternberg, 2003). Following this opinion, contemporary literature defines mathematical giftedness in terms of the individual's abilities in complex problem solving (Koshy, Ernest, & Casey, 2009; Sriraman, 2005; Wiczerkowski, Croplay, & Prado, 2000).

Krutetskii's (1976) seminal study introduces components of high mathematical ability in school children, including the ability to grasp formal structures, think logically in spatial, numeric, and symbolic relationships, generalize rapidly and broadly and be flexible with mental processes. According to Krutetskii, students with high mathematical ability appreciate clarity, simplicity, and rationality and can be characterized by the general synthetic component called mathematical cast of mind. Krutetskii's study introduced batteries of tests that facilitated analysing different qualities of the mathematical mind. However, due to the multiplicity of the tests and complex research procedures employed, Krutetskii's study led to a broad and comprehensive view of high mathematical ability, though an operational definition of mathematical giftedness that can be used with school students is missing. Lack of precise definitions of mathematical giftedness and insufficient development of methods for assessment and evaluation of mathematical abilities, in general, and high mathematical abilities, in particular, complicate the work with mathematically talented individuals. To solve the above-mentioned problem, we suggest employing the data and methods of neurocognitive research.

## **1.2 Education and neuroscience**

Since 1998, when Byrnes and Fox suggested that brain research findings may have useful applications for education, many researchers have not only supported this opinion theoretically (and practically to a much smaller degree) in their experimental studies, but have also made an effort to apply the obtained neurocognitive data to the development of educational theory and practice (Byrnes & Fox, 1998; Christodoulou & Gaab, 2009; De Smedt, Ansari, Grabner, Hannula, Schneider, & Verschaffel, 2010; Geake, 2008; O'Boyle

& Gill, 1998; Willingham, 2009; Woolcott, 2011). Note, however, that despite the growing number of neurocognitive research studies dealing with mathematics and individual abilities (mostly investigated separately), there has been far less research in the field of educational neuroscience than is desirable.

### *1.2.1 Brain research of giftedness*

Over the years researchers have carried out several studies designed to investigate the neurophysiologic basis of giftedness, in general, and mathematical giftedness, in particular.

The brains of the mathematically gifted show enhanced development and activation of the right hemisphere (Singh & O'Boyle, 2004; Prescott, Gavrilesu, Cunnington, O'Boyle, & Egan, 2010). Another characteristic of mathematically gifted individuals is enhanced brain connectivity (Jung & Haier, 2007; Geake, 2009; O'Boyle, 2005) and an ability to activate task-appropriate regions in both brain hemispheres in a well-orchestrated and coordinated manner (O'Boyle, 2005). There is also strong evidence of particularly developed prefrontal and posterior parietal regions of the brain (Jung & Haier, 2007; Geake, 2009, Desco, Navas-Sanchez, Sanchez-Gonzales, Reig, Robles, et al., 2011) and enhanced intra-hemispheric fronto-parietal connectivity (Jung & Haier, 2007; Prescott, et al., 2010). Moreover, there is strong empirical evidence that individuals with higher intelligence tend to exhibit lower frontal brain activation compared with individuals with lower intelligence (for review, Neubauer & Fink, 2009). Using electroencephalography (EEG), Gevins and Smith (2000) have shown differences in amplitude measures that correspond to participants' ability. There is a topographic pattern of differences related to the level of general intelligence. That is, the high-ability participants showed greater parietal, and less prefrontal, activity than their low-ability counterparts (Gevins & Smith, 2000; Jaušovec & Jaušovec, 2004). Moreover, highly intelligent participants displayed more brain activity in the early stages of task performance, while average individuals displayed it more in the later stages of task execution (Jaušovec & Jaušovec, 2004).

### *1.2.2 Brain research on mathematics*

A considerable body of research has been conducted towards understanding of the neural foundation of mathematical cognition (Dehaene, Piazza, Pinel, & Cohen, 2003; Santens, Roggeman, Fias, & Verguts, 2010). Brain research literature demonstrates quite consistent findings that associate different mental operations with brain location; that is, memory retrieval is associated with the prefrontal cortex (e.g., Badre & Wagner, 2005) and representation (Danker & Anderson, 2007), verbal encoding (e.g., Clark & Wagner, 2003), mental rotation (e.g., Heil, 2002), and visuo-spatial strategies in mathematics (e.g. Sohn, Goode, Koedinger, Stenger, Fissell, Carter, & Anderson, 2004) are associated with the parietal cortex. However, when complexity of the problems rises, more brain areas simultaneously support the solving process. The difficulty of mathematical problems raises the complexity of the neuro-cognitive mechanisms involved in solving the problems (Zamarian, Ischebeck, & Delazer, 2009). From this point of view, insight-based problems represent one of the most complex types of problem-solving tasks, in general, and mathematical problems, in particular. Solving of these problems is not based on

previous knowledge and is seemingly characterized by different neurocognitive mechanisms involved in the processing of them (e.g., Bowden & Jung-Beeman, 2003). Thus, despite a growing number of neurocognitive studies in the field of mathematical processing and individual differences (including high abilities), issues related to neurophysiology of mathematical giftedness are still being investigated insufficiently.

Accordingly, the possibility of using brain research data for identification and evaluation of mathematical giftedness is not considered practically. To the best of our knowledge, in the literature such data, or even proposals of the most general nature, do not exist. Meanwhile, examination of brain activity in various groups of students when they solve mathematical problems of various types can become an essential source of information on individual differences in mathematical ability, and seemingly can be a key to devising effective methods for the detection and evaluation of mathematical giftedness.

## 2. The study: Multidimensional examination of mathematical giftedness

In this paper, we present some partial data obtained in our large-scale multidimensional examination of mathematical giftedness (Leikin & Leikin, 2010). Sampling procedure in the study (described in detail in the Research Sample section) was performed based on two orthogonal characteristics: general giftedness (G) and excellence in mathematics (EM). Batteries of tests were designed to examine brain activity associated with mathematical problem solving (See Research Instrument section). In our previous publications (Leikin, Waisman, Shaul, & Leikin, 2012; Shaul, Leikin, Waisman, & Leikin, 2012; Waisman, Shaul, Leikin, & Leikin, 2012) we examined the effects of G and EM factors by means of particular tests employed in our study. In this paper we generalize the findings from a variety of tests and suggest a new definition of mathematically gifted individuals. The generalized research findings are reflected in the following observations:

- A. Effects of G and EM factors are task-dependent in both behavioral and neurophysiological measures:
  - A1. EM factor has significant main effects mainly in tasks that require implementation of knowledge familiar to students from school mathematics.
  - A2. G factors have significant main effects in insight-based problems which are not part of school mathematical curriculum and, thus, require original mathematical reasoning.
- B. Mathematical performance in gifted students who excel in mathematics (G-EM students) on insight-based tasks has specific characteristics in both behavioral and electrophysiological results.
- C. G-EM participants exhibited superior performance in all the tests with a constant neuro-efficiency effect.

Based on these observations we suggest that mathematically gifted students are those who are both generally gifted and excel in mathematics.

We return to consider these statements in light of the major findings from our study, after introducing the research procedure and methodology employed.

## 2.1 Research sample

To the best of our knowledge, this study is unique in its differentiation between excellence in mathematics (E-factor) and general giftedness (G-factor). Correspondingly, a sample of about 200 students was chosen from a population of 1200 10<sup>th</sup>-11<sup>th</sup> grade students (16-18 years old). All participants shared a similar socio-economic background and were paid volunteers.

### 2.1.1 G factor – General giftedness

Participants in G groups were mainly chosen from classes of gifted students (IQ>130). The entire research population was examined using Raven's Advanced Progressive Matrix Test (RPMT) (Raven, Raven, & Court, 2000). RPMT serves as a good indicator of general intelligence and the ability to decompose problems into manageable segments (Carpenter, Just, & Shell, 1990; Raven, et al., 2000; Silverman, 2009). We used a shortened Raven test containing 30 items (range varies from 0 to 30) (adopted from Koichu, 2003). Each item has a 3 x 3 matrix with a missing cell. For each item, the participant selects the best option from among eight possible answers which completes the given matrix. Items are presented in black characters on a white background, and become increasingly difficult to solve as the subject progresses through each set. The test time limit is 15 minutes. Following the test, students with a Raven score of higher than 27 who do not study in a gifted class were designated as the gifted sample.

### 2.1.2 EM factor – Excellence in mathematics

All 1200 students studied mathematics at high and regular levels. Note that mathematics is a compulsory subject in Israeli high schools and students can be placed in one of three levels of mathematics: high, regular and low. The level of instruction is determined by students' mathematical achievements in earlier grades. The instruction at HL (high level) differs from that at RL (regular level) in terms of the depth of the learning material and the complexity of the mathematical problem-solving involved. The items we used in our study are basic items for both the RL and HL curriculum and are learned identically by students in both groups.

Students who were included in the sample as excelling in mathematics studied mathematics at HL with scores higher than 90. Additionally, excellence in mathematics was examined with the SAT-M (Scholastic Aptitude Test in Mathematics) test. SAT-M is a multi-component test that includes tasks from different realms of mathematics. SAT-M is a recognized tool for measuring academic abilities in mathematics (e.g., Young & Kobrin, 2001) and strategic problem-solving skills (e.g., Morrison, Collymore, Saul, & Paul, 2006) and having high correlations with IQ tests (Benbow, Lubinski, Shea, & Eftekhari-Sanjani, 2000).

We used a short version of an SAT-M test containing 35 items with a time limit of 30 minutes (adopted from Zohar, 1990). The range varied from 0 to 35 and represented the number of problems solved correctly. Students who received an SAT-M score above 26 constituted 2% of the 1200 students in the research population and thus were chosen as a control measure for the sampling procedure.

At the end of the sampling procedure our target population consisted of four groups of participants according to the following combinations of E and G factors:

G-EM group: students who are identified as generally gifted and excelling in mathematics;

G-NEM group: students who are identified as generally gifted but do not excel in mathematics;

NG-EM group: students excelling in mathematics who are not identified as generally gifted;

NG-NEM group: students who are identified as neither generally gifted nor excelling in mathematics.

Overall, 200 students participated in this study. In this paper we use data collected for 117 male students as presented in Table 1.

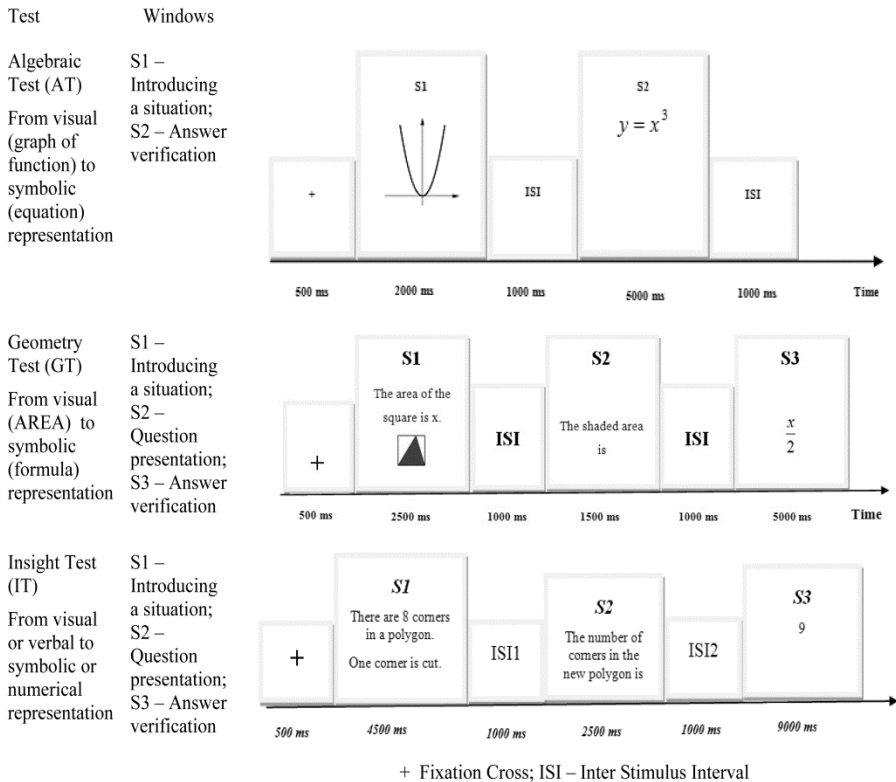
**Table 1:**  
Research sample

	<b>G-EM</b>	<b>G-NEM</b>	<b>NG-EM</b>	<b>NG-NEM</b>	<b>Total</b>
Boys	29	30	24	26	117

## 2.2 Research Instrument

### 2.2.1 ERP Technique

We used the ERP (Event-Related Brain Potentials) technique that offers a high temporal resolution in the course of problem solving due to a precise reflection of perceptive and cognitive mechanisms. ERPs are electrophysiological measures reflecting changes in the electrical activity of the central nervous system related to external stimuli or cognitive processes occurring in the brain. These measures provide information about the process in real time, before the appearance of any external response (Neville, Coffey, Holcomb, & Tallal, 1993). ERP waves are known as components and are classified according to four criteria: polarity (P-positive and N-negative); wave strength (amplitude); time of appearance following the stimulus (latency); and distribution across the scalp (Gaillard, 1988; Halgren, 1990). Different ERP components [named according to polarity and time of appearance, e.g., N (negative) 100 or P (positive) 300] are thought to be related to different cognitive processes, such as early perceptual stages of stimuli processing or



**Figure 1:**  
Examples of the item design in the three tests selected for this paper.

stimulus evaluation and classification (e.g., Nittono, Nageishi, Nakajima, & Ullsperger, 1999; Oades, Dittmann-Balcar, & Zerbin, 1997). The ERP technique has useful applications in language-related research (e.g., Kaan, 2007) and recently was adapted for the study of creativity, solving of insight-based problems and mathematical processing (e.g., Dietrich & Kanso, 2010).

To examine brain activity associated with solving mathematical problems we designed tests corresponding to school mathematical curriculum and to the skills necessary for success in mathematical problem solving. As the basis for task design we chose translation between different representations of mathematical objects (visual, symbolic, verbal and numerical) and their different combinations. Additionally, an insight-based test was especially designed in order to analyse brain activity associated with solving unfamiliar problems. Overall, we designed nine tests of which six proved to be reliable (Cronbach's Alpha >0.68). In this paper, we use three of these tests, which are comparable from the point of view of translations between the visual and symbolic representations that they



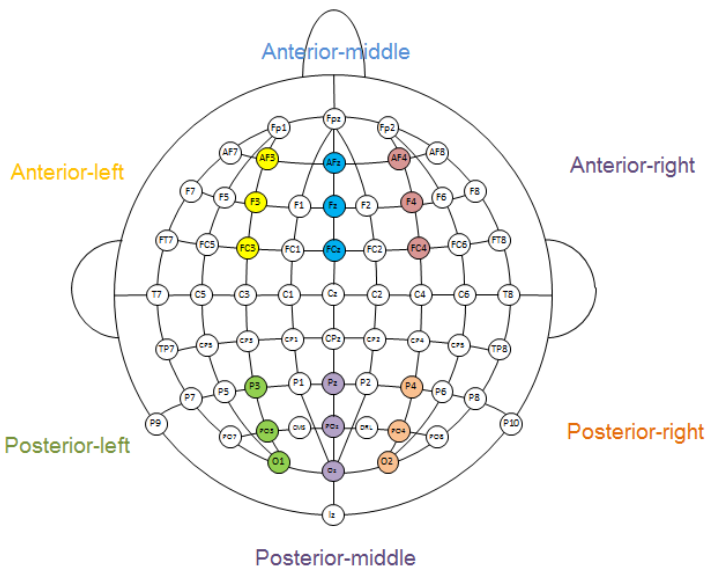
require. The selected tests are: Geometry tests (GT) that included area-related problems, Algebraic tests (AT) that included tasks of translation between visual and symbolic representations of functions, and Insight-based tests (IT) (Figure 1).

Computerized tests were designed and administered using E-Prime software (Schneider, Eschmann, & Zuccolotto, 2002). Each test included 60 tasks (trials). All tasks were presented in the center of the computer screen and displayed in black characters on a white background.

Each task on the AT was presented in two windows that appeared consecutively: S1 – introduction of task condition, and S2 – answer verification (see Figure 1). Each task on the GT and IT was presented in three windows with different stimuli (S1 – presenting a situation stage, S2 – question presentation stage and S3 – answer verification stage) that appeared consecutively (see Figure 1).

2.2.2 Data recording procedure

Participants are seated 105 cm away from the computer screen in a sound attenuated room. Scalp EEG data are continuously recorded using a 64 channel BioSemi ActiveTwo system (BioSemi, Amsterdam, The Netherlands) and ActiveView recording software. Pin-type electrodes are mounted on a customized Biosemi head-cap, arranged according to the 10–20 system. Two flat electrodes are placed on the sides of the eyes in order to monitor horizontal eye movement. A third flat electrode is placed underneath the left eye



**Figure 2:**  
Location of the electrodes.

to monitor vertical eye movement and blinks. During the session electrode offset is kept below 50  $\mu\text{V}$ . The EEG signals are amplified and digitized with a 24 bit AD converter. A sampling rate of 2048 Hz (0.5 ms time resolution) is employed. Figure 2 depicts location of the electrodes.

## 2.3 Data analysis

### 2.3.1 Behavioral data analysis

We examined Accuracy and Reaction time for correct responses (RTc) for each participant. Accuracy was determined by the percentage of correct responses. Reaction time for correct responses (RTc) was the mean RTc for all trials of the test. Between group differences in RTc and Accuracy were examined with MANOVA for the G factor (2 levels: G and NG) and the EM factor (2 levels: EM and NEM) as a between-subject factor, while RTc and accuracy were examined as two interdependent measures (e.g., Jensen, 2006).

### 2.3.2 Electrophysiological data analysis

We performed the following statistical analysis connected to early components and late potentials.

First, in order to examine early differences in amplitude and latency of early components associated with perceptual processing of each stage we conducted, for each stage, repeated measures of MANOVA using Laterality (3 levels: P, PO, O) as within-subject factor, G factor (2 levels: G, NG) and EM factor (2 levels: EM, NEM) as between-subject factors.

Second, in order to examine the mean overall electrical activity from the whole scalp we performed repeated measures MANOVA on RMS (i.e., the square root of the mean of the squared potentials from each common referenced electrode) using Time (3 levels: 250-500, 500-700 and 700-900 ms) as within-subject factor and G factor (2 levels: G, NG) and EM factor (2 levels: EM, NEM) as between-subject factors.

Third, the mean amplitudes were averaged over six regions (see Figure 2): PR - posterior right (P4, PO4, O2), PM - posterior middle (Pz, POz, Oz), PL - posterior left (P3, PO3, O1), AR - anterior right (AF4, F4, FC4), AM - anterior middle (AFz, Fz, FCz) and AL - anterior left (AF3, F3, FC3). Repeated measures MANOVA was performed on the ERP mean amplitude considering the six electrode sites (PL, PM, PR, AL, AM and AR) as within-subject factors, the G factor (2 levels: G, NG) and EM factor (2 levels: EM, NEM) as between-subject factors. The measures were the mean amplitude in three aforementioned time frames. This was done for each stage of problem solving for each task. This analysis was done in order to ascertain the differences in electrical activity between the experimental groups in predefined parts of the scalp.

Fourth, we conducted repeated measures MANOVA with two orthogonal factors: Caudality (6 levels: AF, F, FC, P, PO and PO), Laterality (3 levels: left, middle, right) as within-subject factors and with G factor (2 levels: G, NG), EM factor (2 levels: EM,

NEM) as between-subject factors. The measures were the mean amplitude in the three aforementioned time frames for each stage of problem solving for each task. This analysis was done in order to examine the differences in the hemispheric dominance between the experimental groups.

Pairwise comparisons were used for further investigation of effects. For all analyses, *p* values were corrected for deviation from sphericity according to the Greenhouse Geisser method.

Table 2 summarizes the statistical analysis of within- and between- subject differences.

**Table 2:**  
Summary of statistical analysis procedure for electrophysiological data

Statistical analysis	Stage	Between-subject factor	Within-subject factor	Measures
P1	S1, S2		Laterality: Left, Middle, Right	Latency and Amplitude at P, PO,O electrodes
P2	S1			Latency and Amplitude at F, FC electrodes
Repeated measures Manova			Three time frames 6 electrode sites: PL, PM, PR, AL, AM and AR	RMS (root mean square) Mean amplitude at each electrode site AL, AM, AR, PL, PM, PR at each of the three time frames
Late potentials	consequent ANOVAs pair-wise comparisons	S1 S2 S3	Laterality: Left, Middle, Right Caudality: AF, F, FC, P, PO, O	Mean amplitude at each of 18 electrodes: AF3, AFz, AF4, F3, Fz, F4, FC3, FCz, FC4, P3, PZ, P4, PO3, POz, PO4, O1, Oz, O4 at each of the three time frames

### 3. Results and discussion

This section aims to demonstrate the power of neurocognitive examination of the activity associated with solving mathematical problems as an evaluation tool that leads to the definition of mathematical giftedness. Our aim here is to provide empirical support for the following observations:

- A. Effects of G and EM factors are task-dependent both in behavioral and neurophysiological measures:
  - A.1 EM factor has significant main effects mainly in tasks that require implementation of knowledge familiar to students from school mathematics.
  - A.2 G factor has significant main effect in the insight-based problems, which are not part of the school mathematical curriculum, and, thus, they require original mathematical reasoning.
- B. Mathematical performance in gifted students who excel in mathematics (G-EM students) on insight-based tasks has specific characteristics in both behavioral and electrophysiological results.
- C. G-EM participants exhibited superior performance in all the tests with a constant neuro-efficiency effect.

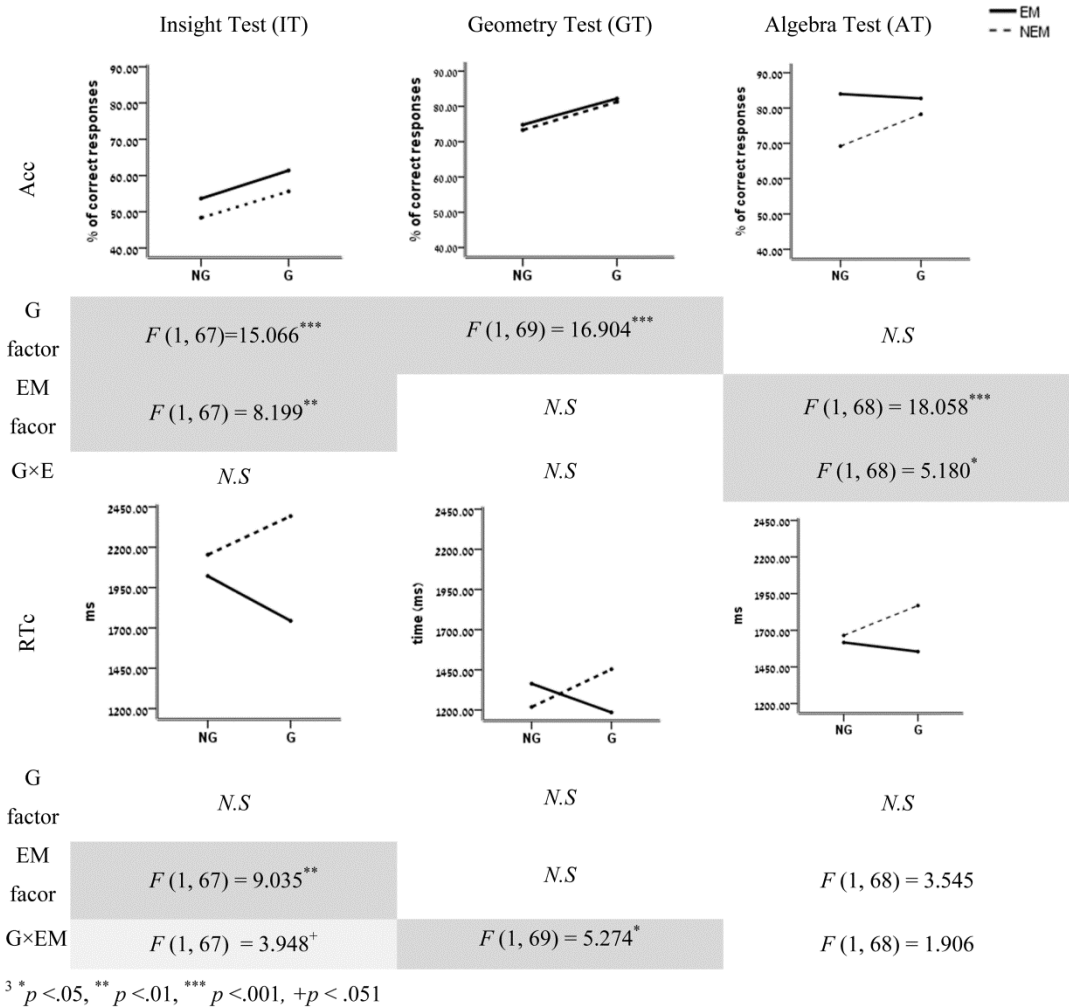
Based on these observations, we suggest that mathematically gifted students are those who are both generally gifted and excel in mathematics.

#### 3.1 Behavioral evidence

We found that G and EM factors had different effects on Accuracy and RTc in the three tests presented here. The EM factor had a significant effect on accuracy on insight-based and function-related tests, whereas the G factor significantly influenced accuracy on insight-based and area-related tests (see Figure 3). Notably, both G and EM factors had a significant effect on accuracy only on the Insight test.

Generally speaking, participants excelling in mathematics were more accurate and had lower reaction times for correct responses than their non-excelling counterparts on all tests. However, the EM factor had a significant effect on reaction time for correct responses only on the insight-based test; furthermore, a significant interaction between G and EM factors was achieved only for accuracy measures on the function-related test and for RTc (reaction time for correct responses) on the area-related tests (see Figure 3).

Task difficulty appeared to lead to the increase in RTc and decrease in Accuracy (see Figure 3): Insight-based tasks are considered to be the most difficult tasks both from the theoretical (Kershaw & Ohlsson, 2004) and practical (actual familiarity) perspectives. At the same time, areas and functions – the topics included in curriculum and learned in high school – were relatively easier as reflected in participants' improved performance. EM participants were more accurate when performing these tasks and spent less time on production of the correct responses. Increased accuracy was especially significant on the function-related test.



**Figure 3:**

Accuracy and Reaction Time for correct responses in the four experimental groups.

Thus, EM and G factors seem to have different effects in different mathematical tasks, or, in other words, performance (accuracy and RTc) of G and EM students is task dependent. The results demonstrate that the EM factor influenced performance on insight-based and function-related tests. It appears, however, that G-EM participants had the highest accuracy and the lowest RTc scores on all three tests. G-NEM demonstrated high accuracy along with slightly high reaction time for correct responses as compared to NG-EM participants. On the basis of these data we can make at least four important assumptions.

First, the G factor has a significant main effect on the insight-based problems, which are not part of school mathematical curriculum, while the EM factor has a significant main effect mainly in tasks that require implementation of knowledge familiar to students from school mathematics.

Second, the EM factor attains complete realization only in interaction with the G factor; that is, the G-EM group outperforms all the other groups.

Third, the G-NEM participants invest more time in order to provide correct responses. This suggests that giftedness has a strong impact on the NEM group's performance. In turn, this suggests that to some extent giftedness can compensate for lack of excellence.

Fourth, excellence in mathematics and giftedness seem to be interrelated but independent factors. Moreover, mathematical giftedness appears to be an independent phenomenon that emerges from the combination of two factors: EM and G.

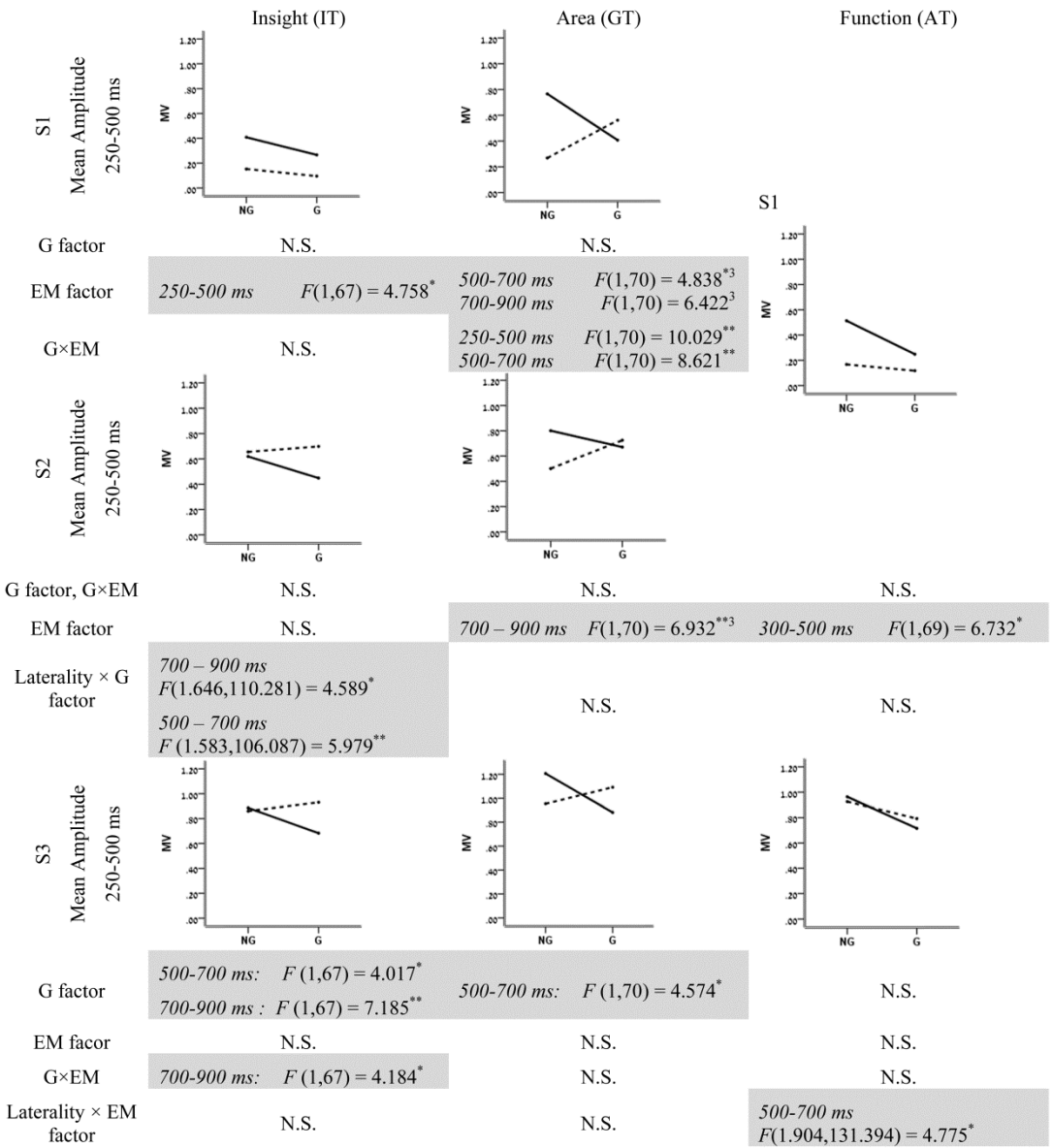
### 3.2 Neurocognitive evidence

In addition to behavioral measures we found support for our hypotheses in electrophysiological measures. Further to the findings in the behavioral data, we revealed different main effects of between-subject factors on electrophysiological measures in tests with different task demands (see Figure 4). The electrophysiological measures (amplitudes) which related to G and EM factors show quite a different picture on the different tests.

The G and EM factors influence brain activity differently in various tests. In tests that include the material learned in school (such as area- and function-related tests), the brain activity of EM and NEM significantly differs mostly within the NG group. The EM individuals within the NG group (as compared to NEM ones) displayed higher electrical activation, which is an indicator of investments of high cognitive effort. Within the G group no significant differences were found between EM and NEM participants. However, G-EM subjects displayed the lowest (but not significantly so) electrical activity as compared to the other experimental groups. The aforementioned phenomena were similar in all stages of tests that included tasks that are part of the school curriculum. In contrast, there was no significant difference between NG-EM and NG-NEM on insight-based tasks at the stages of question presentation and answer verification. The significant difference between EM and NEM emerged within the G group only. This demonstrates that neural efficiency within the EM group exists only in combination with the G factor.

Notably, the interaction between laterality and EM factor presents in tests connected with school learning programs, whereas the interaction between laterality and G factor presents in tests that are not based on the school curriculum.

First, the electrophysiological differences in between-subject factors were found at different stages of task problem solving. On the insight test the differences were reflected only at the second (question presentation) and third (answer verification) stages, while all groups processed the first (introducing a situation) stage in the same way. Quite a different picture was obtained on the area-related task, where the differences between the



<sup>1</sup> EM – Excelling, G – Gifted, NEM – non Excelling, NG – non Gifted, <sup>2</sup> \*  $p < .05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$ , +  $p < .51$ , N.S. – No significant effects are found, <sup>3</sup> RMS

**Figure 4:**  
The electrophysiological effects at late potential stages.

groups were found at all three stages of task performance. In contrast, in the function-related task the differences among the four experimental groups were achieved only at the first (introducing a situation) stage. Thus, it may be suggested that different participant groups vary in processing of particular stages of different tasks. That is, the effects of the G and EM factors were task-dependent in neurophysiological measures, too, and these effects were expressed by differences in brain activation at different processing stages.

Second, G and EM factors influenced brain activation patterns differently in different tasks. On the insight-based task the G factor was found to be significant at S3 (i.e., answer verification stage). The results also revealed significant interaction between hemispheric laterality and the G factor at S2 (i.e., question presentation stage) which was more prominent in the later parts of the time course. More specifically, whereas generally gifted participants demonstrated higher mean amplitudes in the right brain hemisphere, NG participants activated the left hemisphere more strongly. The differences in activation patterns between gifted and non-gifted participants seem to relate to distinctions between the respective processing strategies used by them when solving insight-based problems. The literature suggests that brain hemispheres function differently in processing varying types of information and that they contribute differently to solving different types of problems (Fiore & Schooler, 1998). Accordingly, we suggest that G students not only start to solve insight-based problems at the question presentation stage (when differences first appear), but they also more significantly utilize the right hemisphere regions for this purpose. Note also that mathematically gifted children have been found to activate their right hemisphere much more than their non-gifted peers even when solving non-insight problems (O'Boyle, et al., 2005; Prescott, et al., 2010). Thus, gifted individuals seem to be more successful in insight-based problem solving, in particular, because they utilize for this purpose more appropriate processing strategies and brain mechanisms.

The electrophysiological measures (amplitudes) related to G and EM factors show quite a different picture on different tests. First, the electrophysiological differences in between-subject factors were found at different stages of task problem solving. In insight-based tasks the differences were reflected only at the second (question presentation) and third (answer verification) stages, while all groups processed the first (introducing a situation) stage in the same way. Quite a different picture was obtained on the area-related task. Here, the differences between the groups were expressed at all three stages of the task. In contrast, on the function-related task the differences among the four experimental groups were achieved only at the first (introducing a situation) stage. Thus, it may be suggested that different groups vary in the processing of particular stages of different tasks. That is, effects of G and EM factors were task-dependent in neurophysiological measures too, and these effects were expressed by differences in brain activation at the different processing stages.

In contrast, on the area-related task, analysis of brain potentials revealed a main effect of EM factor at S1 (introducing a situation stage) and at S2 (the question presentation stage). Excelling in mathematics participants had higher overall mean activity than their non-excelling counterparts. However, pairwise comparisons revealed that these differ-



ences were significant only among the non-gifted participants. The mean amplitude in the NG-EM group was higher than in the NG-NEM group. However, among gifted participants the mean amplitude in EM and NEM groups was similar. This effect was most prominent at the middle posterior (PM) electrode site. Therefore, we can conclude that G-NEM and G-EM participants process the stage of introducing a situation (geometric figure with shaded area) and the next stage (question presentation) in the same way. Accordingly, it may be suggested that, at least at S1 (introducing a situation) stage of area-related tasks, the EM factor loses its impact among the G participants. In other words, for successful solving of some mathematical problems, general giftedness seems to have a greater impact than excellence in mathematics. These data are in line with the above presented suggestions:

- A. Effects of G and EM factors are task-dependent;
- B. G factor has a significant main effect in the insight-based problems, which are not part of school mathematical curriculum; and
- C. Mathematical performance in gifted students who excel in mathematics (G-EM students) on insight-based tasks has specific electrophysiological characteristics.

Additional confirmation of the differences between G and EM factors were found in the function-related task. The analysis of late brain potentials (ERPs) in this task revealed a significant effect of the EM factor at S1 (introducing a situation stage). EM participants exhibited higher mean amplitudes than NEM participants. These differences were the most prominent at middle posterior brain sites and were significant among NG individuals.

## Discussion

Participants in our study solved three types of mathematical problems of varying complexity. The area- and function-related tasks are based on the school curriculum and are taught in high school classes. On the other hand, the insight-based problems are not part of the school curriculum and are considered to be relatively difficult to solve (Kershaw & Ohlsson, 2004). These problems are usually unfamiliar to solvers and require high cognitive effort associated with discovering new ways of solution, even though the knowledge for their solution has been previously learned by the solver (Mayer, 1995; Mullis, et al., 2003). Seemingly as a result, performance on insight-based tasks is characterized by low accuracy rates and high reaction times for correct responses. In contrast, the learning appropriate topics in high school usually improved students' performance quality (i.e., increased accuracy rates and decreased reaction times) as was demonstrated on the area- and function-related tasks.

The above noted differences in the nature of the three types of problems integrated in our study (learned in school vs. not learned in school) were expressed in separate main effects of EM or G factors. The EM factor mainly related to the tasks based on school curriculum (area- and function-related problems). In turn, the G factor appears to be significant for the solving of non-familiar problems that require insight. Moreover, on

the insight-based tasks, the G factor was also expressed in hemispheric dominance (holistic matter) and not in specific electrode locations (locale matter) with a tendency towards right brain hemisphere dominance.

G students also demonstrated lower overall mean amplitudes for correct responses at the answer verification stage in all three types of mathematical problems considered in this study. That is, they seem to exhibit more efficient brain activation patterns during processing of this cognitive task (Haier, Siegel, Nuechterlein, Hazlett, et al., 1988; Neubauer & Fink, 2009). The effect of G factor was especially significant in insight-based and area-related tasks and not in function-related tasks.

The most efficient brain functioning, accompanied by highest behavioral performance, was observed in the G-EM group. The most prominent differences between G-EM and the other three groups were shown on the insight-based tasks. As in all tests, the mean amplitude of G-NEM participants does not significantly differ from NG-EM and NG-NEM participants. The compensation for the lack of EM by the G factor, as exhibited in the G-NEM group appears in all tests; however, on the insight-based test this compensation was the most prominent. The mean amplitude of G-EM individuals was found to be the lowest compared to the other three groups. In contrast, the NG-EM participants exhibited the highest mean amplitude as compared to the other participant groups (with good behavioral performance). So it seems that non-gifted but excelling in mathematics students demonstrate the largest cortical effort when dealing with the presented tasks. Accordingly, it may be suggested that the neural efficiency (low cortical arousal) is not connected solely to excellence in mathematics (EM factor). It seems to be connected to G and EM factors together, that is, to mathematical giftedness as a specific cognitive phenomenon. Notably, NG-NEM students forgo investing cortical effort since they might have already reached their limit.

The findings indicate that electrophysiological data support the hypothesis that in solving mathematical problems, the G factor may to some extent compensate for the lack of excellence in mathematics. In contrast, the EM factor cannot compensate for the lack of general giftedness.

In conclusion, mathematically gifted individuals who excel in mathematics (G-EM individuals) exhibit superior performance on all the tests with a constant neuro-efficiency effect. G and EM factors have different effects on the performing of various mathematical tests. Characteristics of brain activity and between-factor relationships depend on the task's demand. Insight-based tests facilitate identification of G-EM individuals on the basis of both behavioral and electrophysiological measures.

## References

- Ardila, A. & Rosselli, M. (2002). Acalculia and Dyscalculia. *Neuropsychology Review*, 12(4), 179-231.
- Badre, D., & Wagner, A. D. (2005). Frontal lobe mechanisms that resolve proactive interference. *Cerebral Cortex*, 15, 2003–2012.

- Benbow, C. P., Lubinski, D., Shea, D. L., & Eftekhari-Sanjani, H. (2000). Sex differences in mathematical reasoning ability: Their status 20 years later. *Psychological Science, 11*, 474-480.
- Berg, D.H. (2008). Working memory and arithmetic calculation in children: The contributory roles of processing speed, short-term memory, and reading. *Journal of Experimental Child Psychology, 99*, 288-308.
- Bowden, E. M., & Jung-Beeman, M. J. (2003). Aha! Insight experience correlates with solution activation in the right hemisphere. *Psychonomic Bulletin & Review, 10*, 730-737.
- Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology, 19*, 273-293.
- Butterworth, B. (1999). *The Mathematical Brain*. London: MacMillan.
- Byrnes, J.P., & Fox, N.A. (1998). The educational relevance of research in cognitive neuroscience. *Educational Psychology Review, 10* (3), 297-342.
- Callahan, C.M. (2000). Intelligence and giftedness. In R. J. Sternberg (Ed.), *Handbook of Intelligence* (pp. 159-175). Cambridge: Cambridge University Press.
- Carpenter, P. A., Just, M. A. & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Matrices Test. *Psychological Review, 97*, 404-431.
- Christodoulou, J.A., & Gaab, N. (2009). Using and misusing neuroscience in education-related research. *Cortex, 45*, 555-557.
- Clark, D., & Wagner, A. D. (2003). Assembling and encoding word representations: fMRI subsequent memory effects implicate a role for phonological control. *Neuropsychologia, 41*, 304-317.
- Danker, J., & Anderson, J. (2007). The roles of prefrontal and posterior parietal cortex in algebra problem solving: A case of using cognitive modeling to inform neuroimaging data. *Neuroimage, 35*, 1365-1377.
- Davidson, J. E. & Sternberg, R. J. (2003). *The Psychology of Problem Solving*. Cambridge, UK: Cambridge University Press.
- Davis, G.A., & Rimm, S.B. (2004). *Education of the Gifted and Talented*. MA: Allyn & Bacon.
- De Smedt, B., Ansari, D., Grabner, R.H., Hannula, M.M., Schneider, M., & Verschaffel, L. (2010). Cognitive neuroscience meets mathematical education. *Educational Research review, 5*, 97-105.
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*, 487-506.
- Desco M, Navas-Sanchez, F.J., Sanchez-González, J., Reig, S., Robles, O., Franco, C., Guzmán-De-Villoria, J.A., García-Barreno, P., Arango, C. (2011). Mathematically gifted adolescents use more extensive and more bilateral areas of the fronto-parietal network than controls during executive functioning and fluid reasoning tasks. *Neuroimage, 57*(1), 281-292.

- Dietrich A., & Kanso R. (2010). A review of EEG, ERP, and neuroimaging studies of creativity and insight. *Psychological Bulletin*, *136*(5), 822-48.
- Feldman, H.D. (2003). A developmental, evolutionary perspective of giftedness. In Borland, J.H. (Ed.). *Rethinking Gifted Education* (pp. 9-33). New York: Teachers College Press.
- Fiore, S. M., & Schooler, J. W. (1998). Right hemisphere contributions to creative problem solving: Converging evidence for divergent thinking. In M. Beeman & C. Chiarello (Eds.), *Right hemisphere language comprehension: Perspectives from cognitive neuroscience* (pp. 349-371). Mahwah, NJ: Erlbaum.
- Gagné, F. (1985). Giftedness and Talent: Reexamining a Reexamination of the Definitions. *Gifted Child Quarterly*, *29*(3), 103-112.
- Gaillard, A. W. K. (1988). Problems and paradigms in ERP research, *Biological Psychology*, *26*, 91-109.
- Gardner, H. (1983/2003). *Frames of mind: The theory of multiple intelligences*. New York: Basic.
- Geake, J. (2008). Neuromythologies in education. *Educational Research*, *50*(2), 123-133.
- Geake, J. G. (2009). *The brain at school: Educational neuroscience in the classroom*. London: McGraw-Hill.
- Gear, D.C. (1993). Mathematical disabilities: Cognitive, Neuropsychological, and genetic components. *Psychological Bulletin*, *114*, 345-362.
- Gevins, A., & Smith, M. E. (2000). Neurophysiological measures of working memory and individual differences in cognitive ability and cognitive style. *Cerebral Cortex*, *10*, 830-839.
- Haier, R.J., Siegel, B.V., Nuechterlein, K.H., Hazlett, E., Wu, J.C., Paek, J., Browning, H.L., Buchsbaum, M.S. (1988) Cortical glucose metabolic rate correlates of abstract reasoning and attention studied with positron emission tomography. *Intelligence*, *12*, 199–217.
- Halgren, E. (1990). Insights from evoked potentials into the neuropsychological mechanisms of reading. In: A.B. Scheibel & A.F. Wechsler (Eds.) *Neurobiology of Higher Cognitive Function*. NY: Guilford Press.
- Heil, M. (2002). The functional significance of ERP effects during mental rotation. *Psychophysiology*, *39*, 535–545.
- Hoard, M.K., Gear, D.C., Byrd-Craven, J., & Nugent, L. (2008). Mathematical cognition in intellectually precocious first graders. *Developmental Neuropsychology*, *33*(3), 251–276.
- Jaušovec, N., & Jaušovec, K. (2000). Correlations between ERP parameters and intelligence: A reconsideration. *Biological Psychology*, *55*, 137-154.
- Jaušovec, N., & Jaušovec, K. (2004). Differences in induced brain activity during the performance of learning and working-memory tasks related to intelligence. *Brain and Cognition*, *54*, 65–74.
- Jensen, A. R. (2006). *Clocking the mind: Mental chronometry and individual differences*. Amsterdam, the Netherlands: Elsevier.
- Jung, R.E., & Haier, R.J. (2007). The parieto-frontal integration theory (P-FIT) of intelligence: Converging neuroimaging evidence. *Behavioral and Brain Sciences*, *30*, 135–154.

- Kaan E. (2007). Event-Related Potentials and Language Processing: A Brief Overview. *Language and Linguistics Compass*, 1(6), 571–591.
- Kershaw, T. C. & Ohlsson, S. (2004). Multiple causes of difficulty in insight: The case of the nine-dot problem. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 30, 3-13.
- Kershaw, T.C., & Ohlsson, S. (2004). Multiple causes of difficulty in insight: The case of the nine-dot problem.
- Koshy, V., Ernest, P., & Casey, R. (2009). Mathematically gifted and talented learners: Theory and practice. *International Journal of Mathematical Education in Science and Technology*, 40 (2), 213-228.
- Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. Translated from Russian by Teller, J.; Edited by Kilpatrick J. & Wirszup, Chicago: The University of Chicago Press.
- Leikin, R. & Leikin, M. (2010). *Multidimensional examination of mathematical genius as compared to mathematically gifted and non gifted students*. Research proposal granted by the John Templeton Foundation.
- Leikin, R., Koichu, B. & Berman A. (2009). Mathematical giftedness as a quality of problem-solving acts. In R. Leikin, A. Berman & B. Koichu (Eds), *Creativity in mathematics and the education of gifted students*. (Ch. 8, pp. 115-127). Rotterdam, the Netherlands: Sense Publisher.
- Leikin, R., Waisman, I., Shaul, S., & Leikin, M. (2012). Solving short insight-based problems: An ERP study with gifted and excelling male adolescents. *The Proceedings of the 36<sup>th</sup> International Conference for Psychology in Mathematics Education (V. 3, pp. 82-90)*. Taiwan: Taipei.
- Marland, S. P., Jr. (1972). *Education of the Gifted and Talented*. Washington, DC: US Government Printing Office.
- Mayer, R. E. (1995). The search for insight: Grappling with Gestalt psychology's unanswered questions. In R. J. Sternberg & J. E. Davidson (Eds.), *The nature of insight* (pp. 3–32). Cambridge, MA: MIT Press.
- Meyer, M. L., Salimpoor, V. N., Wu, S. S., Geary, D. C., & Menon, V. (2010). Differential contribution of specific working memory components to mathematics achievement in 2<sup>nd</sup> and 3<sup>rd</sup> graders. *Learning and Individual Differences*, 20(2), 101-109.
- Morrison, W., Collymore, R., Saul, M. & Paul, K. (2006). *Remember to read the question: A thinking students' guide to the SAT and beyond*. Dubuque, IO: Kendall & Hunt.
- Mullis, I. V. S., Martin, M. O., Smith, T. A., Garden, R. A., Gregory, K. D., Gonzalez, E. J., et al. (2003). *TIMSS assessment frameworks and specifications 2003* (2nd ed). Chestnut Hill, MA: Boston College.
- Neubauer, A.C., & Fink, A. (2009). Intelligence and neural efficiency. *Neuroscience and Biobehavioral Reviews*, 33, 1004–1023.
- Neville, H. J., Coffey, S.A., Holcomb, P.J., & Tallal, P. (1993). The neurobiology of sensory and language processing in language-impaired children. *Journal of Cognitive Neuroscience*, 5, 235-253.

- Nittono H, Nageishi Y, Nakajima Y, Ullsberger P (1999) Event-related potential correlates of individual differences in working memory capacity. *Psychophysiology*, 36, 745–754.
- O’Boyle, M.W., & Gill, H. S. (1998). On the relevance of research in cognitive neuroscience to educational practice. *Educational Psychology Review*, 10(4), 397-409.
- Oades, R.D., Dittmann-Balcar, A., & Zerbin, D. (1997). Development and topography of auditory event-related potentials (ERPs): Mismatch and processing negativity in individuals 8-22 years of age. *Psychophysiology*, 34, 677-693.
- O’Boyle, M. W. (2005). Some current findings on brain characteristics of the mathematically gifted adolescent. *International Educational Journal*, 6(2), 247-251.
- Piirto, J. (1999). Identification of the academically talented: High-IQ Talent and specific academic talent. In J. Piirto, *Talented children and adults: Their development and education*. (ch. 3, pp.90-135). Upper Saddle River, N.J.: Prentice Hall.
- Prescott, J., Gavrilesco, M., Cunnington, R., O’Boyle, M. W., & Egan, G. F. (2010). Enhanced brain connectivity in math-gifted adolescents: An fMRI study using mental rotation. *Cognitive Neuroscience*, 1(4), 277–288.
- Raven J., Raven J.C. & Court J.H. (2000). *Manual for Raven’s Progressive Matrices and Vocabulary Scales*. Oxford: Oxford Psychologists.
- Santens S., Roggeman, C., Fias, W., & Verguts, T. (2010) Number processing pathways in human parietal cortex. *Cerebral Cortex*, 20,77–88
- Shaul, S., Leikin, M. Waisman, I., & Leikin, R. (Accepted). Visual processing in algebra and geometry in mathematically excelling students.: an ERP study. To be presented at the TSG-16 (Topic Study Group-16: Visualization in mathematics education), *the 12th International Congress on Mathematics Education*, July, 2012, Coex, Seoul, Korea.
- Silverman, L. K. (2009). The measurement of giftedness. In L.V. Shavinina (ed.), *International Handbook on Giftedness* (pp. 947-970), New York: Springer.
- Singh, H. & O’Boyle, M.W. (2004). Differences in interhemispheric interaction during global/local processing in mathematically gifted adolescents, average ability youth and college students. *Neuropsychology*, 18, 371-377.
- Sohn, M.-H., Goode, A., Koedinger, K. R., Stenger, V. A., Fissell, K., Carter, C. S., & Anderson, J. R. (2004). Behavioral equivalence, but not neural equivalence – neural evidence of alternative strategies in mathematical thinking. *Nature Neuroscience*, 7(11), 1193.
- Sriraman, B. (2005). Problem solving as a pre-cursor to Mathematical Proof. *Mathematics in School*, 34(1), 4-8.
- Sternberg, R. J. (Ed.) (2000). *Handbook of Intelligence*. New York, NY: Cambridge University Press.
- Swanson, H. L. (2006). Cognitive processes that underlie mathematical precociousness in young children. *Journal of Experimental Child Psychology*, 93, 239–264.
- Taub, G. E., Floyd, R. G., Keith, T. Z., & McGrew, K. S. (2008). Effects of general and broad cognitive abilities on mathematics achievement. *School Psychology Quarterly*, 23 (2), 187–198.

- Waisman, I., Shaul, S., Leikin, M. & Leikin, R. (2012) (Accepted). An ERP study on geometric reasoning in excelling in mathematics adolescents. To be presented at symposium Cognitive and Neuro-Cognitive Research on High Mathematical Abilities at the ECHA 2012 Conference.
- Wieczerkowski, W., Croplay, A.J., & Prado, T. M. (2000). Nurturing Talents/Gifts in mathematics. In K.A. Heller, R. J. S. F.J. Monks & R.F. Subotnik (Eds.), *International handbook of giftedness and talent* (2nd ed., pp. 413-425). Oxford: Elsevier.
- Willingham, D.T. (2009). Three problems in the marriage of neuroscience and education. *Cortex*, 45, 544-545.
- Winner, E. (2000). Giftedness: Current theory and research. *Current Directions in Psychological Science*, 9(5), 153-156.
- Woolcott, G. (2011). Mathematics and giftedness: Insight from educational neuroscience. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [New] Practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia) and the Australian Association of Mathematics Teachers) (pp.830-838). Adelaide: AAMT and MERGA.
- Young, J. W. & Kobrin, J. L. (2001). *Differential validity, differential prediction, and college admission testing: A comprehensive review and analysis*. Research report No. 2001-6 (Published online). Retrieved October 10, 2005, from: [http://www.collegeboard.com/repository/differential\\_validity\\_10539.pdf](http://www.collegeboard.com/repository/differential_validity_10539.pdf)
- Zamarian, L., Ischebeck, A., & Delazer, M. (2009). Neuroscience of learning arithmetic: Evidence from brain imaging studies. *Neuroscience and Biobehavioral Reviews*, 33, 909-925.
- Ziegler, A. & Raul T. (2000). Myth and reality: A review of empirical studies on giftedness. *High Ability Studies*, 113-136.
- Zohar, A. (1990). *Mathematical reasoning ability: its structure, and some aspects of its genetic transmission*. Unpublished Doctoral Dissertation, Hebrew University, Jerusalem.