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Empirical evaluation of the near-miss-to-Weber's law: a visual discrimination experiment

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Abstract

Many pure tone intensity discrimination data support the hypothesis that the sensitivity function grows as a power function of the stimulus intensity (near-miss-to-Weber's law). In order to test whether the near-miss-to-Weber's law fits empirical data from other sensory modalities than hearing, the participants of the experiment had to compare the perceived area of squares presented on a computer screen. The results indicate an almost perfect fit of the near-miss-to-Weber's law, which is in line with many pure tone intensity discrimination data. Different from a recent study on psychoacoustics, however, the exponent in the near-miss-to-Weber's law does not vary with the criterion value used to define "just-noticeably different". Furthermore, we provide evidence that, for a majority of the participants, Weber's classical law provides an equally good fit to the data as the near-miss-to-Weber's law.

Key words: Weber's law; near-miss-to-Weber's law; psychophysical model; size judgment; size discrimination; empirical evaluation

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1. Introduction

The term "near-miss-to-Weber's law" refers to small but systematic deviations from Weber's classical law, which is usually expressed as

$$\Delta(a) = ca. \tag{1}$$

Whereas Equation (1) postulates that the Weber fraction $\frac{\Delta(a)}{a}$ is a constant function of the stimulus intensity a, many pure tone intensity discrimination data indicate that the Weber fraction decreases with the stimulus intensity. Although already Fechner (1860/1889) observed systematic deviations from Equation (1), a successful modification of Weber's law was not proposed before 1932: By reanalyzing empirical data on judgments of line lengths, Guilford (1932) came to the conclusion that the power law

$$\Delta(a) = ca^{\alpha} \tag{2}$$

with an exponent α slightly less than 1, fits empirical data better than Weber's original law. Although many empirical data in psychoacoustics confirm this result (e.g., Green, Nachmias, Kearney & Jeffress, 1979; Hanna, von Gierke & Green, 1986; Jesteadt, Wier & Green, 1977; McGill & Goldberg, 1968a, 1968b; Neff & Jesteadt, 1996; Penner, Leshowitz, Cudahy & Ricard, 1974; Schacknow & Raab, 1973; Schroder, Viemeister & Nelson, 1994; Viemeister & Bacon, 1988), a recent paper by Doble, Falmagne and Berg (2003) questions the validity of Guilford's power law on logical grounds.

To appreciate their arguments, we first have to introduce some notation: To begin with, we note that a central problem with Equations (1) and (2) is that the just-noticeable-difference $\Delta(a)$ is not properly operationalized and therefore, cannot be measured satisfactorily. To avoid these difficulties, today, Weber's law and its modifications are usually expressed in terms of choice- or discrimination-probabilities: In the following, let $P_a(x) \equiv p(x,a)$ denote the probability that a comparison stimulus of the intensity x is perceived as being more intense than a standard stimulus of the intensity a. Then, for a fixed standard stimulus a, the function $x \mapsto P_a(x)$ is referred to as *psychometric function*. It is worth mentioning that the standard stimulus[3] a and the comparison stimulus x belong to different observation areas (Dzhafarov, 2002, 2003). In a paired-comparison experiment, for instance, the standard stimulus a may be presented first, followed by the comparison stimulus x (or vice versa). Similarly, if we are dealing with visual objects, then a may be presented to the left and x to the right of a fixation mark (Dzhafarov, 2002). Alternatively, the procedural status of the standard stimulus a and the comparison stimulus x may differ. For instance, if an adaptive procedure is used in an experiment, then the standard stimulus a may be held constant throughout a block of trials, and the comparison stimulus x may be changed according to the responses of the subject. In that case, $p(x,a)$ can be interpreted as the probability that x is perceived as being more intense than a, when stimulus a is held fixed in a block of trials.

[3] In the following, we do not distinguish between the physical stimulus and its physical intensity.

In the following, the psychometric function $x \mapsto P_a(x)$ is assumed to be strictly increasing with the stimulus intensity x (e.g., Augustin, 2008; Doble, Falmagne, & Berg, 2003, Falmagne, 1985, 1994; Iverson, 2006). Consequently, the sensitivity function ξ can be defined as $\xi_\pi(a) := P_a^{-1}(\pi)$, where a is a fixed standard stimulus and π denotes a discrimination probability in the open unit interval $(0,1)$. Then, by definition, $\xi_\pi(a)$ is that stimulus intensity which is judged greater than a with probability π: $P_a(\xi_\pi(a)) = \pi$. Thus, the just-noticeable-difference $\Delta_\pi(a)$ can be operationalized as[4] $\Delta_\pi(a) := \xi_\pi(a) - a$, which suggests the following modification of Guilford's power law:

$$\Delta_\pi(a) = C(\pi)a^{\alpha(\pi)}, \tag{3}$$

where $C(\pi)$ and $\alpha(\pi)$ are real valued parameters that may depend on the value of the criterion π (Doble, Falmagne & Berg, 2003).

Recently, Doble et al. (2003) pointed out that Guilford's power law with an exponent $\alpha(\pi) \neq 1$ is in conflict with the balance condition $p(a,x) + p(x,a) = 1$: If Guilford's power law holds along with the balance condition, then the exponent $\alpha(\pi)$ in Equation (3) is necessarily a constant equal to 1 (Doble et al., 2003, Theorem 1). A problem arises from the fact that the balance condition is frequently enforced in the analysis of the empirical data by averaging over experimental conditions. For details see Doble et al. (2003) and Augustin (2004).

For this reason, Doble et al. (2003) advocate a slightly modified model which postulates that, for any criterion value π, there exist real valued constants $K(\pi)$ and $\beta(\pi)$ such that for any stimulus intensity a,

$$\xi_\pi(a) = K(\pi)a^{\beta(\pi)}. \tag{4}$$

It is important to note here that Equation (4) plays a crucial role in the modelling of intensity discrimination data for the following reasons: First and foremost, there is great empirical evidence in favor of Equation (4), since it fits the same data as Guilford's power law (3) (Doble et al., 2003, Thesis 2). Furthermore, an advantage of the near-miss-to-Weber's law, as compared to Guilford's power law, is that the balance condition is consistent with a non-constant parameter function $\pi \mapsto \beta(\pi)$ in Equation (4) (e.g., Augustin, 2008, Theorem 4). Furthermore, it is worth mentioning that the exponent $\beta(\pi)$ provides a measure of deviation from Weber's law $\Delta_\pi(a) = C(\pi)a$. Note that, according to Equation (4),

$$\Delta_\pi(a) = \xi_\pi(a) - a = K(\pi)a^{\beta(\pi)} - a = (K(\pi)a^{\beta(\pi)-1} - 1)a,$$

which shows that the deviations from Weber's law depend on the value of the exponent $\beta(\pi) - 1$. If, for instance, $\beta(\pi) = 1$ for all criterion values π, then Weber's law results:

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[4] Note that we do not assume that $\xi_{0.5}(a) = a$. Therefore, $\Delta_\pi(a)$ can also be defined as $\Delta_\pi(a) := \xi_\pi(a) - \xi_{0.5}(a)$. However, we know of no argument why the one definition should be preferred to the other.

$$\Delta_\pi(a) = (K(\pi) - 1)a. \tag{5}$$

For this reason, we refer to Equation (4) as the *near-miss-to-Weber's law*.

It is worth mentioning that, by definition, the parameter function $\pi \mapsto K(\pi)$ in Equation (4) is strictly increasing with the criterion value π (cf. Doble, Falmagne, Berg, & Southworth, 2006): First of all, we note that the underlying physical stimulus scale can always be transformed in such a way that, for instance, the smallest standard stimulus is denoted by 1 (Augustin, 2008). Furthermore, by definition, $\pi \mapsto \xi_\pi(1)$ is an increasing function of π, and according to Equation (4), $\xi_\pi(1)$ equals $K(\pi)$ in the near-miss-to-Weber's law.

Similarly, Doble et al. (2003) argued that, for theoretical reasons, the parameter function $\pi \mapsto \beta(\pi)$ is a non-constant function of the criterion value π, provided that the balance condition $p(a,x) + p(x,a) = 1$ is satisfied empirically (Doble et al., 2003, Thesis 3).

Psychoacoustical data by Doble, Falmagne, Berg, and Southworth (2006) provide some evidence that the parameter functions $\pi \mapsto \beta(\pi)$ and $\pi \mapsto K(\pi)$ in the near-miss-to-Weber's law are monotonically decreasing and increasing, respectively. Additionally, the data indicate that $K(\pi)$ and $\beta(\pi)$ are related according to

$$K(\pi) = \varepsilon\left(\frac{1}{\delta}\right)^{\beta(\pi)}, \tag{6}$$

where the parameters ε and δ correspond to tone intensities near the upper threshold of hearing. According to this co-variation, the near-miss-to-Weber's law can be written as

$$\xi_\pi(a) = \varepsilon\left(\frac{a}{\delta}\right)^{\beta(\pi)}, \tag{7}$$

with a non-constant parameter function $\pi \mapsto \beta(\pi)$. Doble et al. (2006) emphasize the following fixed point property of Equation (7): For any criterion value π, and any intensity a,

$$\begin{aligned}\ln\left(\frac{\xi_\pi(a)}{a}\right) &= \ln(\varepsilon) - \beta(\pi)\ln(\delta) + (\beta(\pi) - 1)\ln(a)\\ &= \ln(\varepsilon) - \ln(a) + \beta(\pi)(\ln(a) - \ln(\delta)),\end{aligned} \tag{8}$$

which shows that for *any* value of the criterion π, the graphs of $\ln(\frac{\xi_\pi(a)}{a})$ versus $\ln(a)$ intersect at the common point $(\ln(\delta), \ln(\varepsilon) - \ln(\delta))$. Doble et al. (2006) argue that this is in accordance with the idea of a high-level fixed point which is used in the subjective evaluations of the intensities of acoustical stimuli. A similar result was found by Stevens (1974): If, for a given sensory modality, the experimental conditions (e.g., level of adaption, stimulus duration, inhibitory stimulation,...) are varied, then the resulting psychophysical power functions intersect at a common point near the respective ceiling level.

It is noteworthy that there is also theoretical evidence in favor of Equation (7): Iverson (2006), for instance, argues that according to the assumptions of Fechnerian psychophysics, the near-miss-to-Weber's law can either be specialized into Weber's law (5), or Equation (7) with $\varepsilon = \delta$. Aczél and Falmagne (1999) show that the parameters in Equation (4) are necessarily related according to Equation (6) provided that the exponent $\beta(\pi)$ is a non-constant function of the criterion value π and a Fechnerian representation of the form $p(a,x) = F(u(a) - g(x))$ exists. Finally, Augustin (2008) discusses the near-miss-to-Weber's law in the context of psychometric models of discrimination. He assumes that Equation (4) holds with a non-constant exponent $\beta(\pi)$. If, additionally, a representation of the form $P_a(x) = F(\rho(a)x^{\gamma(a)})$ exists, then the near-miss-to-Weber's law holds in the form of Equation (7).

The central aim of the present study was to examine whether the near-miss-to-Weber's law (4) fits empirical data from other sensory modalities than hearing. For this purpose, we performed a visual discrimination experiment: participants had to decide which of two squares appeared to be the larger one. Furthermore, we tested whether the parameter function $\pi \mapsto \beta(\pi)$ is a non-constant function of the criterion value π, as was found by Doble et al. (2006) for intensity discrimination data. To this end, five different values of the criterion π (0.2, 0.375, 0.5, 0.625, and 0.8) were used in the experiment. Finally, it was tested whether the near-miss-to-Weber's law can be replaced by Weber's law (5), which postulates that the exponent in the near-miss-to-Weber's law is a constant equal to 1.

2. Method

2.1 Participants

Fourteen students at Graz University participated in the experiment. The median age of the sample, consisting of nine women and five men, was 24 years (range, 21-26). All participants had normal or corrected-to-normal visual acuity, and none of them had previous experience in visual discrimination tasks.

2.2 Stimuli and apparatus

The experiment was performed in a soundproof cabin, which was fully darkened. The only source of illumination came from the monitor (21"EIZO FlexScan Color LCD). In order to keep the distance to the monitor constant, the participant's head was stabilized by a chin-rest. The stimuli, presented on the monitor, consisted of green and blue squares of the same saturation and luminance. The green stimulus was the referent (80, 90, 100, 110, 120 pixel side length[5]) and the blue comparison stimulus was adaptively adjusted using the weighted up-down method by Kaernbach (1991). The participants entered their responses by pressing

[5] which corresponds to 2.7, 3.0, 3.3, 3.7, and 4.0 cm on the screen.

one of two buttons on a standard keyboard (CHERRY G83). The software used for the presentation of the stimuli and the recording of the responses was Orange 4.6b.[6]

2.3 Procedure

The experiment consisted of two sessions separated by a day. The procedures were similar on both days: All participants were tested individually. At the beginning of each session, the participant had to sit down at a desk with a keyboard and an LCD monitor on it. The participant's head was supported by a chin-rest at a distance of 75 cm from the screen. The investigation started with a written instruction on the screen, followed by a practice block, which continued until four direction reversals (turn points) were obtained. The data collection was divided into five blocks, with two-minute breaks between Blocks 1 and 2, and Blocks 4 and 5, and breaks of five minutes between Blocks 2 and 3, and Blocks 3 and 4.

Each trial proceeded as follows: a blue and a green square were presented simultaneously on a dark-grey background and the participant had to report – via keyboard – which of the two squares was the larger one (cf. Figure 1). There was no time limit for responding and no feedback was given. A new trial was initiated immediately after a response. On each trial, the referent and the comparison stimulus appeared randomly at non-overlapping positions on the monitor, such that (i) both squares were entirely visible, (ii) the edges of the squares were parallel to the edges of the screen, and (iii) the minimal distance between the squares was greater than 30 pixel (1 cm on the screen).

Figure 1:
Stimulus configuration displayed on the monitor. In this trial, the left square was the referent and the right square was the comparison stimulus. In the experiment, the referent was presented in green and the comparison stimulus in blue.

[6] which was developed by M. D. Kickmeier-Rust, University of Graz.

Each of the five different referents (a = 80, 90, 100, 110, 120 pixel side length) was combined with each of the five different criterion values (π = 0.2, 0.375, 0.5, 0.625, 0.8), resulting in 25 independent adaptive tracks per person. Within any block, the criterion value π was fixed, and the referent stimuli varied: We used the referents with 80, 100, and 120 pixel side lengths within Session 1, and the referents with 90 and 110 pixel side lengths within Session 2 (cf. Table 1). In order to ensure variability, the referent was chosen at random at the beginning of each trial within any block. In both sessions, the criterion values were arranged in the following order: 0.375 (Block 1) – 0.2 (Block 2) – 0.5 (Block 3) – 0.8 (Block 4) – 0.625 (Block 5). Consequently, there were three adaptive tracks per block within Session 1, and two adaptive tracks per block within Session 2.

Each adaptive track continued until 30 direction reversals (turn points) were obtained. Then a threshold estimate – in the following denoted as $\xi_\pi(a)$ – was computed for each track by arithmetic averaging the side lengths of the squares at the observed turn points, excluding the first four. It is important to note here that, according to the simulation studies by García-Pérez (2000), threshold estimates should be based on at least 30 direction reversals. Different from Doble et al. (2006), the size of the comparison stimulus was adaptively adjusted using Kaernbach's (1991) weighted up-down method[7], which is a version of the simple up-down procedure, following the rule of decreasing the comparison stimulus after a positive response (which, in our experiment, means that the comparison stimulus is perceived as being greater than the referent), and increasing the comparison stimulus after a negative response. However, whereas the simple up-down procedure uses equal step sizes for upward and downward steps, the weighted up-down method allows the step size for upward steps (Δ_{up}) to differ from the step size for downward steps (Δ_{down}). A crucial

Table 1:

Schedules for the tracks in Sessions 1 and 2. The referents in Session 1 were squares with 80, 100, and 120 pixel side length. In Session 2, squares with 90 and 110 pixel side length were presented. For each criterion value π , the step sizes Δ_{up} and Δ_{down} were computed according to Equation (9).

Session 1:	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**
Referents	80,100,120	80,100,120	80,100,120	80,100,120	80,100,120
π	0.375	0.2	0.5	0.8	0.625
Δ_{up}	3	3	3	12	5
Δ_{down}	5	12	3	3	3
Session 2:	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**
Referents	90,110	90,110	90,110	90,110	90,110
π	0.375	0.2	0.5	0.8	0.625
Δ_{up}	3	3	3	12	5
Δ_{down}	5	12	3	3	3

[7] Doble et al. (2006) used Levitt's (1971) transformed up-down procedures to estimate the 16%-, 21%-, 29%-, 71%-, 79%-, and 84%-performance levels on the psychometric function.

advantage of this method as compared to Levitt's (1971) transformed up-down procedure is that the weighted up-down method can converge to any desired performance level on the psychometric function. If the upward step size Δ_{up} is related to the downward step size Δ_{down} according to

$$\frac{\Delta_{up}}{\Delta_{down}} = \frac{\pi}{1-\pi}, \tag{9}$$

then the weighted up-down procedure converges to the value $\xi_\pi(a)$, which is judged greater than the referent a with probability π (Kaernbach, 1991, 2001). For instance, in order to target the 62.5% level on the psychometric function, the ratio of upward step size Δ_{up} and downward step size Δ_{down} has to be equal to

$$\frac{\Delta_{up}}{\Delta_{down}} = \frac{0.625}{0.375} = \frac{5}{3}.$$

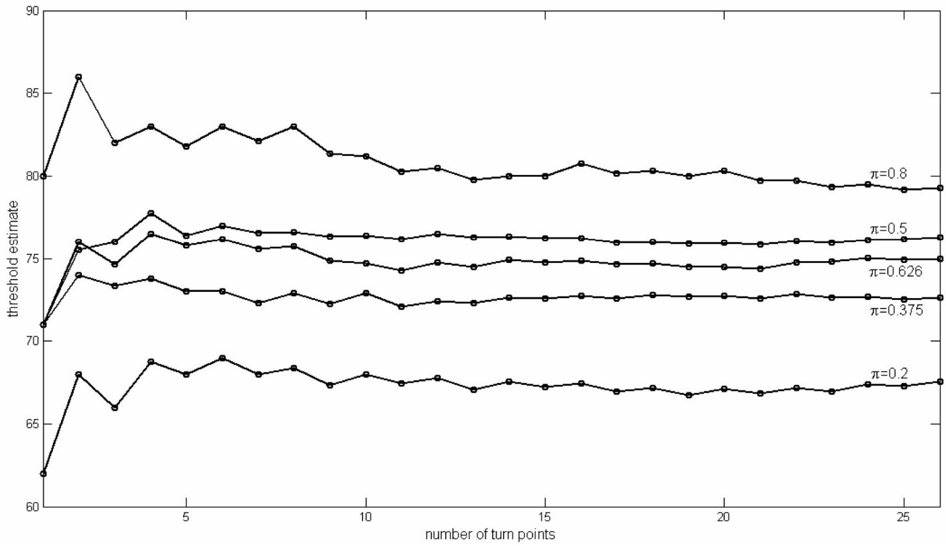
This means an increase of 5 pixel side length (about 0.17 cm) after a negative response and a decrease of 3 pixel side length (about 0.1 cm) after a positive response. Similarly if the upward step size $\Delta_{up} = 5$ and the downward step size $\Delta_{down} = 3$ are interchanged, then the adaptive track converges to the 37.5% level on the psychometric function. Table 1 contains the upward and downward step sizes for all criterion values and the exact schedules for the tracks in both sessions.

3. Results

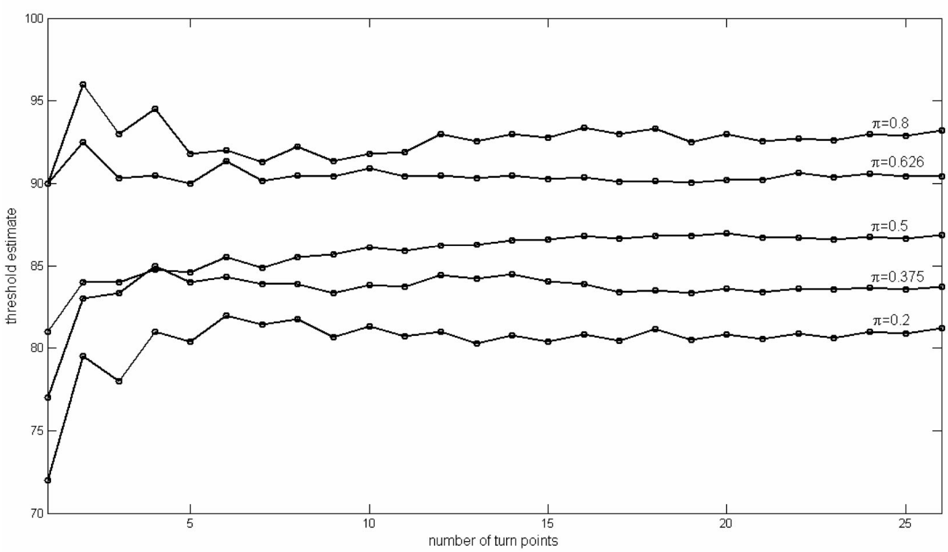
Session 1 took an average time of about 45 minutes and Session 2 of about 30 minutes. None of the participants had to be excluded from the statistical analysis due to missing data or a drop out of the experiment.

First of all, we checked whether 26 turn points are sufficient to provide stable threshold estimates. To this end, let $\xi_\pi^{(n)}(a)$ denote the arithmetic mean of the side lengths at the first n turn points of an adaptive track ($1 \le n \le 26$). Then, for each standard stimulus a and each criterion value π, the value $\xi_\pi^{(26)}(a)$ corresponds to the final threshold estimate $\xi_\pi(a)$. Figure 2, which refers to a single participant (MM04) representative of the whole sample, shows how the threshold estimates $\xi_\pi^{(n)}(a)$ depend on the number n of direction reversals. It can be seen that, with only a few exceptions, the estimates stabilize at about 15-20 reversals, which indicates that an increase of n would have only minor effects on the threshold estimates.

Standard *a*=80



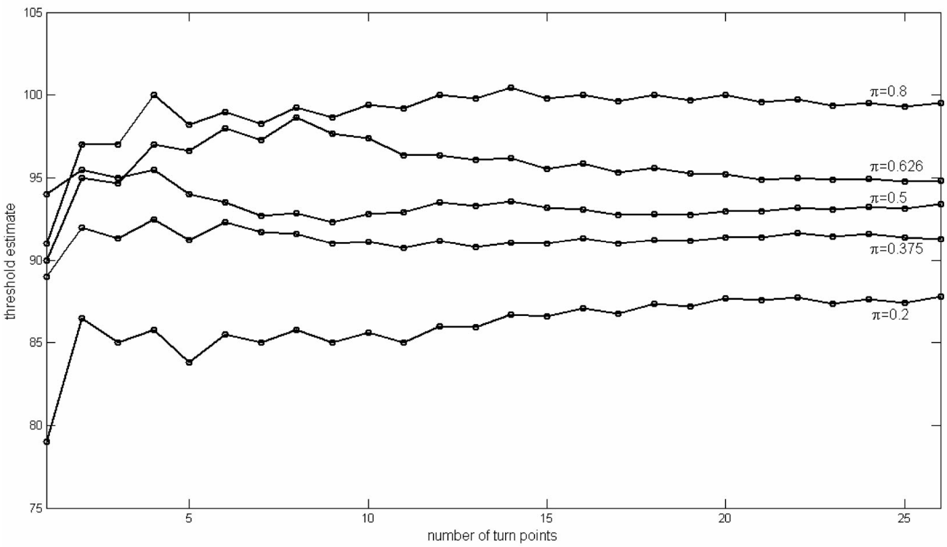
Standard *a*=90



continued →

Figure 2:
Plots of the threshold estimates $\xi\,\pi^{(n)}$ (a) versus n for a single participant (MM04) representative of the whole sample.

Standard a=100



Standard a=110

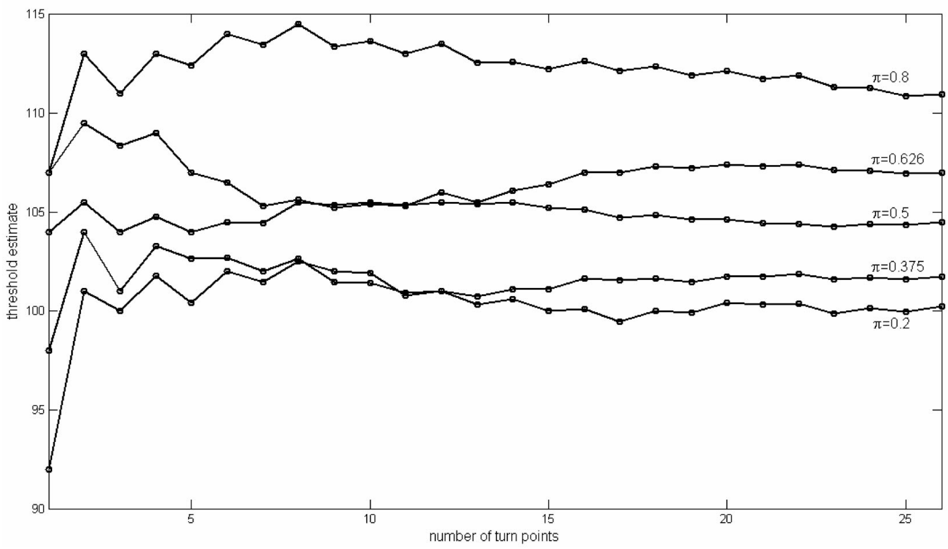
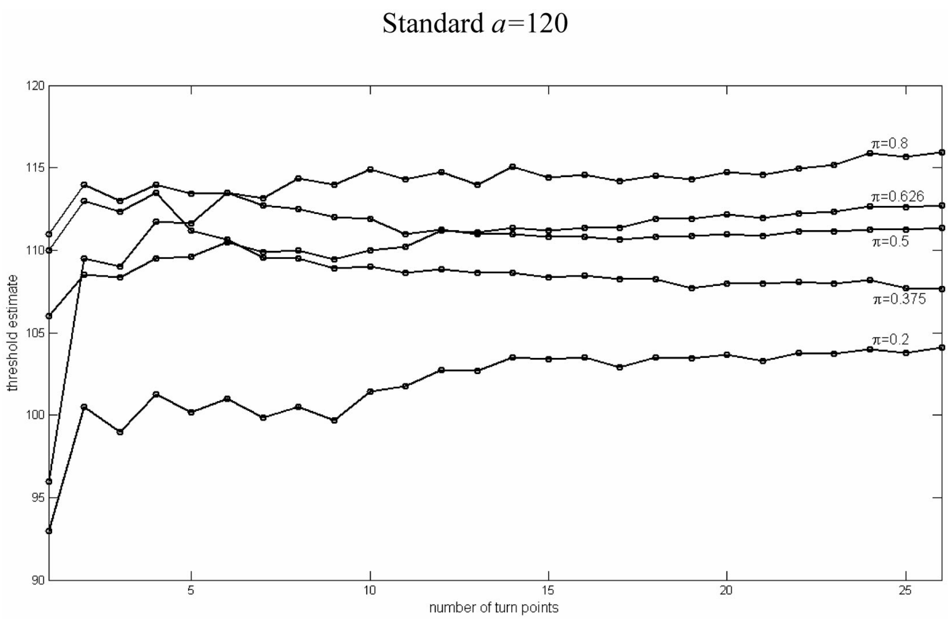


Figure 2 continued →



To test whether the exponent $\beta(\pi)$ in the near-miss-to-Weber's law is a constant function of the criterion value π, we fitted three nested regression models for each participant. The most general model rests upon the assumption that both parameter functions in Equation (4) vary with the criterion value π. To estimate the unknown model parameters, we introduced an *indicator function* 1_p,

$$1_p(\pi) := \begin{cases} 1, & \pi = p, \\ 0, & \pi \neq p, \end{cases}$$

where p and π are real valued numbers in the open unit interval (0,1). Then the parameters $\beta(\pi)$ and $K(\pi)$ in Equation (4) can be estimated from the following regression model:

$$\begin{aligned} \ln(\xi_\pi(a)) = c_0 + c_1 1_{0.375}(\pi) + c_2 1_{0.5}(\pi) + c_3 1_{0.625}(\pi) + c_4 1_{0.8}(\pi) + \\ + c_5 1_{0.2}(\pi)\ln(a) + c_6 1_{0.375}(\pi)\ln(a) + c_7 1_{0.5}(\pi)\ln(a) + \\ + c_8 1_{0.625}(\pi)\ln(a) + c_9 1_{0.8}(\pi)\ln(a). \end{aligned} \tag{10}$$

It is important to note that Equation (10) is equivalent to the following system of linear regressions:

$$\ln(\xi_{0.2}(a)) = c_0 + c_5 \ln(a),$$

$$\ln(\xi_{0.375}(a)) = (c_0 + c_1) + c_6 \ln(a),$$
$$\ln(\xi_{0.5}(a)) = (c_0 + c_2) + c_7 \ln(a),$$
$$\ln(\xi_{0.625}(a)) = (c_0 + c_3) + c_8 \ln(a),$$
$$\ln(\xi_{0.8}(a)) = (c_0 + c_4) + c_9 \ln(a).$$

Since the near-miss-to-Weber's law can be rewritten as

$$\ln(\xi_\pi(a)) = \ln(K(\pi)) + \beta(\pi)\ln(a),$$

we obtain the following parameter estimates:

$$K(0.2) = \exp(c_0), \quad K(0.375) = \exp(c_0 + c_1), \quad K(0.5) = \exp(c_0 + c_2),$$
$$K(0.625) = \exp(c_0 + c_3), \quad K(0.8) = \exp(c_0 + c_4),$$
$$\beta(0.2) = c_5, \quad \beta(0.375) = c_6, \quad \beta(0.5) = c_7, \quad \beta(0.625) = c_8, \quad \beta(0.8) = c_9.$$

Table 2 contains the squared correlation coefficients for all computed regressions and summarizes the individual parameter estimates. The correlation coefficients, ranging from 0.921 to 0.999 with a median of 0.990, indicate a remarkably good model fit. Although this provides strong empirical evidence in favor of Equation (4), we nevertheless note that a considerable number of K estimates is in conflict with the predictions of the near-miss-to-Weber's law. Note that according to Equation (4), $K(\pi)$ equals $\xi_\pi(1)$, and thus, the K estimates in Table 2 are expected to increase with the criterion value π.

The second regression model rests upon the assumption that the exponent in the near-miss-to-Weber's law is a constant function of π. Since this is equivalent to the assumption that the regression coefficients $c_5,\ldots,c_9$ are identical ($c_5 = c_6 = c_7 = c_8 = c_9$), we fitted the following regression model to our data:

$$\ln(\xi_\pi(a)) = c_0 + c_1 1_{0.375}(\pi) + c_2 1_{0.5}(\pi) + c_3 1_{0.625}(\pi) + c_4 1_{0.8}(\pi) + c_5 \ln(a). \tag{11}$$

Table 3 contains the squared correlation coefficients for all computed regressions and summarizes the individual parameter estimates. Similar to Table 2, the correlation coefficients (ranging from 0.918 to 0.997 with a median of 0.984) indicate a remarkably good fit of the model to the data. Furthermore, except for a few cases (JS13: $K(0.5) > K(0.625)$; KK10: $K(0.5) > K(0.625)$; MN01: $K(0.375) > K(0.5)$), the K estimates are strictly increasing with the criterion value π, which is in line with the predictions of the near-miss-to-Weber's law.

Table 2:
Estimated values of the parameters in the near-miss-to-Weber's law (4). The estimates were obtained by fitting a separate linear regression for each participant (cf. Equation (10)). The squared correlation coefficients are denoted as R^2, with the adjusted R^2 in brackets.

Part.	R^2	$K(0.2)$	$K(0.375)$	$K(0.5)$	$K(0.625)$	$K(0.8)$
		β(0.2)	β(0.375)	β(0.5)	β(0.625)	β(0.8)
AL21	.995 (.992)	1.081	1.266	1.400	1.251	1.367
		0.964	0.937	0.923	0.951	0.941
AO46	.979 (.966)	0.986	0.877	1.215	1.172	2.260
		0.970	1.006	0.945	0.959	0.826
CU08	.991 (.985)	0.616	0.966	1.133	0.843	1.937
		1.073	0.992	0.960	1.031	0.859
DE06	.957 (.931)	0.354	0.422	0.580	0.934	2.146
		1.169	1.148	1.087	0.987	0.816
EP08	.988 (.982)	0.790	0.721	1.376	0.847	2.115
		1.015	1.044	0.912	1.023	0.832
FR02	.998 (.996)	0.505	1.214	1.060	1.055	1.315
		1.130	0.951	0.982	0.990	0.951
JS13	.999 (.998)	0.802	1.174	0.880	1.440	1.141
		1.038	0.961	1.028	0.920	0.979
KK10	.921 (.874)	1.474	0.679	0.746	0.668	0.962
		0.878	1.052	1.042	1.065	1.002
MM04	.981 (.969)	0.623	1.134	1.127	1.077	1.453
		1.075	0.953	0.932	0.975	0.918
MN01	.993 (.989)	0.495	0.963	1.028	1.019	1.456
		1.115	0.988	0.973	0.983	0.914
MP28	.978 (.965)	0.251	0.796	0.699	0.868	1.324
		1.251	1.026	1.058	1.013	0.937
SH02	.994 (.990)	1.151	1.353	1.797	1.004	1.926
		0.946	0.924	0.867	0.998	0.866
UG11	.983 (.972)	0.699	1.102	1.141	1.399	1.451
		1.051	0.964	0.964	0.922	0.924
VP67	.996 (.994)	1.171	1.174	1.683	1.130	2.204
		0.948	0.954	0.882	0.972	0.834

Table 3:
Estimated values of the parameters in Equation (13). The estimates were obtained by fitting a separate linear regression for each participant (cf. Equation (11)). The squared correlation coefficients are denoted as R^2, with the adjusted R^2 in brackets.

Part.	R^2	$K(0.2)$	$K(0.375)$	$K(0.5)$	$K(0.625)$	$K(0.8)$	β
AL21	.995 (.993)	1.190	1.231	1.277	1.295	1.353	0.943
A046	.975 (.969)	1.125	1.182	1.236	1.273	1.331	0.941
CU08	.986 (.982)	0.935	1.007	1.017	1.050	1.095	0.983
DE06	.944 (.929)	0.638	0.688	0.715	0.728	0.761	1.041
EP08	.982 (.978)	0.994	1.037	1.077	1.105	1.146	0.965
FR02	.994 (.992)	0.913	0.966	0.972	1.005	1.046	1.001
JS13	.997 (.996)	1.021	1.050	1.071	1.069	1.108	0.985
KK10	.918 (.896)	0.810	0.830	0.874	0.869	0.938	1.008
MM04	.978 (.972)	1.007	1.046	1.080	1.097	1.142	0.971
MN01	.989 (.986)	0.863	0.933	0.931	0.966	1.005	0.995
MP28	.970 (.963)	0.612	0.690	0.702	0.710	0.763	1.057
SH02	.991 (.989)	1.296	1.376	1.407	1.435	1.503	0.920
UG11	.981 (.976)	1.040	1.096	1.136	1.147	1.201	0.965
VP67	.993 (.991)	1.345	1.384	1.429	1.446	1.499	0.918

Furthermore, in order to test whether the near-miss-to-Weber's law can be replaced by Weber's classical law (which postulates an exponent β equal to one), we set the parameter c_5 equal to unity, resulting in the following regression model:

$$\ln(\xi_\pi(a)) = c_0 + c_1 1_{0.375}(\pi) + c_2 1_{0.5}(\pi) + c_3 1_{0.625}(\pi) + c_4 1_{0.8}(\pi) + \ln(a). \quad (12)$$

For a summary of the results see Table 4. It is noteworthy that the squared correlation coefficients, ranging from 0.916 to 0.997 with a median of 0.984, are similar to those reported in Tables 2 and 3. Furthermore, except for participant JS13 ($K(0.5) > K(0.625)$), KK10 ($K(0.5) > K(0.625)$), and MN01 ($K(0.375) > K(0.5)$), the K estimates are strictly increasing with the criterion value π.

Finally, we analyzed whether the model complexity (i.e., the number of parameters in the model) has an influence on the model fit. It is important to note here that the regression models (10) to (12) are nested within each other, that is, for any two of them, one model can be derived from the other by imposing restrictions on the parameter values. In order to test whether these restrictions exhibit a significant lack of fit, we performed pairwise likelihood ratio tests. The results indicate that, for the majority of the participants (9 out of 14), Weber's law provides an equally good fit to the data as the near-miss-to-Weber's law, whereas only for five participants (AL21, FR02, JS13, SH02, VP67), the near-miss-to-Weber's law is superior to Weber's law (cf. Table 5). Furthermore, we note that only for two participants (FR02, JS13), the data provide evidence in favor of a non-constant parameter function $\pi \mapsto \beta(\pi)$.

Table 4:
Estimated values of the parameters in Equation (14). The estimates were obtained by fitting a separate linear regression for each participant (cf. Equation (12)). The squared correlation coefficients are denoted as R^2 , with the adjusted R^2 in brackets.

Part.	R^2	$K(0.2)$	$K(0.375)$	$K(0.5)$	$K(0.625)$	$K(0.8)$
AL21	.992 (.991)	0.918	0.949	0.984	0.998	1.043
A046	.975 (.970)	0.857	0.901	0.942	0.970	1.014
CU08	.986 (.983)	0.864	0.931	0.940	0.970	1.012
DE06	.939 (.927)	0.771	0.832	0.864	0.880	0.919
EP08	.982 (.979)	0.847	0.883	0.918	0.942	0.976
FR02	.994 (.993)	0.916	0.969	0.974	1.008	1.049
JS13	.997 (.996)	0.953	0.980	1.000	0.998	1.035
KK10	.916 (.900)	0.840	0.861	0.907	0.901	0.972
MM04	.978 (.974)	0.880	0.914	0.944	0.959	0.998
MN01	.989 (.987)	0.842	0.910	0.908	0.942	0.980
MP28	.965 (.958)	0.796	0.898	0.914	0.923	0.992
SH02	.987 (.984)	0.898	0.953	0.974	0.994	1.041
UG11	.981 (.977)	0.885	0.932	0.967	0.976	1.022
VP67	.988 (.985)	0.923	0.949	0.980	0.992	1.028

Table 5:
Pairwise comparisons of Equation (12) with Equation (11) (second column), (11) with (10) (third column), and (12) with (10) (fourth column). The table entries are the p values of the computed likelihood ratio tests. Significant results are printed in bold-face.

Part.	(12) vs. (11)	(11) vs. (10)	(12) vs. (10)
AL21	**.003**	.966	.143
A046	.131	.662	.481
CU08	.558	.161	.216
DE06	.517	.395	.471
EP08	.283	.147	.156
FR02	.976	**.001**	**.003**
JS13	.275	**.011**	**.013**
KK10	.916	.945	.978
MM04	.415	.691	.715
MN01	.832	.129	.194
MP28	.236	.299	.281
SH02	**.001**	.238	**.011**
UG11	.299	.728	.771
VP67	**.001**	.069	**.001**

4. Discusssion

The aim of the present study was to examine whether the near-miss-to-Weber's law fits empirical data from other sensory modalities than hearing. To this end, we performed a visual discrimination experiment in which participants had to decide which of two squares appeared to be the larger one. The squared correlation coefficients in Table 2, ranging from $.921$ to $.999$ with a median of 0.990, indicate an almost perfect fit of the near-miss-to-Weber's law to the empirical data at hand. This is in line with earlier findings reported by Doble et al. (2003) and Doble et al. (2006), who found Equation (4) to hold for pure tone intensity discrimination data.

Different from Doble et al. (2006), however, there is clear empirical evidence that the exponent in the near-miss-to-Weber's law is a constant function of the criterion value π and that, consequently, the near-miss-to-Weber's law can be specified as

$$\xi_\pi(a) = K(\pi)a^\beta. \tag{13}$$

The following arguments justify this conclusion: First of all, the computed likelihood ratio tests indicate that, except for two participants (FR02, JS13), Equation (13) provides an equally good fit to the data as Equation (4). Likewise, the squared correlation coefficients in Table 3 (referring to Equation (13)) are similar to those reported in Table 2 (referring to Equation (4)). Finally, it is important to note that except for three cases (JS13: $K(0.5) > K(0.625)$; KK10: $K(0.5) > K(0.625)$; MN01: $K(0.375) > K(0.5)$), the K estimates in Table 3 (and also Table 4) are in line with the predictions of the near-miss-to-Weber's law: Note that according to Equation (4), $K(\pi)$ equals $\xi_\pi(1)$, which is a strictly increasing function of the criterion value π. The K estimates in Table 2, on the other hand, show a decrease in value from $\hat{K}(0.2)$ to $\hat{K}(0.375)$ (3 out of 14 participants), from $\hat{K}(0.375)$ to $\hat{K}(0.5)$ (4 cases), from $\hat{K}(0.5)$ to $\hat{K}(0.625)$ (10 cases), and from $\hat{K}(0.625)$ to $\hat{K}(0.8)$ (1 case). Since Table 2 rests upon the assumption that the exponent in the near-miss-to-Weber's law is a non-constant function of the criterion value π, we obtain further evidence in favor of a constant parameter function $\pi \mapsto \beta(\pi)$.

It is worth mentioning that finding the exponent $\beta(\pi)$ to be a constant function of the criterion value π is in accordance with the following theoretical result by Augustin (2008), who studied the near-miss-to-Weber's law in the context of psychometric models of discrimination: If the psychometric functions can be represented as $P_a(x) = F(\gamma(a) + \rho(a)x)$, then the near-miss-to-Weber's law can only hold with a constant exponent $\beta(\pi)$ (Augustin, 2008, Theorem 7). It is important to note here that psychometric functions are frequently represented as two-parametric functions $P_a(x) = \psi_{\mu(a),\delta(a)}(x)$, where typically, $\psi_{\mu(a),\delta(a)}$ is a sigmoid function such as the distribution function of the logistic,

$$\psi_{\mu(a),\delta(a)}(x) = \frac{1}{1+\exp(-\delta(a)(x-\mu(a)))},$$

Gaussian,

$$\psi_{\mu(a),\delta(a)}(x) = \frac{1}{\sqrt{2\pi}\delta(a)} \int_{-\infty}^{x} \exp\left(-\frac{(t-\mu(a))^2}{2\delta(a)^2} \right) dt,$$

or Gumbel distribution

$$\psi_{\mu(a),\delta(a)}(x) = 1 - \exp\left(-\exp\left(\frac{x-\mu(a)}{\delta(a)} \right) \right)$$

(Strasburger, 2001; Treutwein, 1995; Treutwein & Strasburger, 1999; Wichmann & Hill, 2001). The crucial point is that all these distribution functions can be represented as $\psi_{\mu(a),\delta(a)}(x) = F(\gamma(a) + \rho(a)x)$, which provides strong theoretical evidence in favor of a constant parameter function $\pi \mapsto \beta(\pi)$ (Augustin, 2008).

Nevertheless, finding the exponent in the near-miss-to-Weber's law to be a constant function of the criterion value π seems to conflict with a theoretical result by Doble et al. (2003, Thesis 3): Doble et al. (2003) showed that the parameter function $\pi \mapsto \beta(\pi)$ in Equation (4) is necessarily a non-constant function of π, if the balance condition $p(a,x) + p(x,a) = 1$ is satisfied empirically. To resolve this conflict, we notice that for $x = a$, the balance condition yields $p(a,a) = 0.5$, which is equivalent to $\xi_{0.5}(a) = a$. Since in 85 out of 90 cases (94.4%), the threshold estimate $\xi_{0.5}(a)$ was less intense than the comparison stimulus a ($\xi_{0.5}(a) < a$), we conclude that the balance condition is violated for the present data set.

Finally, we tested the hypothesis that the exponent in the near-miss-to-Weber's law is a constant equal to 1, which is equivalent to the following reformulation of Weber's classical law:

$$\Delta_\pi(a) = \xi_\pi(a) - a = K(\pi)a - a = (K(\pi) - 1)a. \tag{14}$$

The computed likelihood ratio tests indicate that, for 9 out of 14 participants, Equation (14) provides an equally good fit to the data as the near-miss-to-Weber's law (cf. Table 5). Since, however, for the remaining five participants, the near-miss-to-Weber's law is superior to Weber's law, we cannot generally conclude that the exponent in the near-miss is equal to one.

Finally, let us mention that, in order to avoid effects of fatigue, the experiment was performed on two successive days. For this reason, we tested the consistency of the participants' adjustments in a preliminary study which consisted of two identical sessions separated by a day. The task was similar to that in the main experiment: A blue and a green square were presented simultaneously on a computer screen and the participant had to decide which of the two stimuli was the larger one. The referent (green square) was fixed and the comparison stimulus (blue square) was adaptively adjusted using the weighted up-down procedure. For each criterion value[8] ($\pi = 0.2, 0.375, 0.625$ and 0.8), three different participants were tested resulting in a total sample size of 12. On both days, the selected criterion value was combined with five referents (80, 90, 100, 110, 120 pixel side length), resulting in five adaptive tracks per participant. For each referent, the two sets of 30 turn points resulting from Sessions 1 and 2, were compared by means of a Mann-Whitney U test. Since, in total, five tests were computed for each participant, the type I error rate was adjusted by the Bonferroni correction method, resulting in a significance level of $\alpha = 0.01$. The results indicate that the participants produced identical adjustments in both sessions: For nine out of twelve participants, we obtained no significant test result at all, and for two participants only a single violation was observed. The remaining participant, who produced a total of four violations, was excluded from the main experiment.

To sum up, the present study found the near-miss-to-Weber's law to hold in the field of visual perception, which is in line with pure tone intensity discrimination data reported by Doble et al. (2003) and Doble et al. (2006). Thus, the near-miss-to-Weber's law appears to be a general law that holds across different sensory modalities. Additionally, the paper provides strong evidence that the exponent β in the near-miss-to-Weber's law is independent of the criterion value π used to define “just noticeably different”. This is consistent with the following theoretical result by Augustin (2008): If the near-miss-to-Weber's law holds for a psychometric function of the form $P_a(x) = F(\gamma(a) + \rho(a)x)$, then the exponent in the near-miss-to-Weber's law is necessarily a constant function of the criterion value π. Finally, the data indicate that, for a majority of the participants, Weber's classical law provides an equally good fit to the data as the near-miss-to-Weber's law. Since, however, for five out of 14 participants, the near-miss-to-Weber's law is superior to Weber's law, we cannot generally conclude that β is equal to one.

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[8] Note that, due to a change in the experimental design, the criterion value $\pi = 0.5$ was incorporated only after the preliminary study.

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